КОНТРОЛЬНАЯ РАБОТА №3

Задание.

1. Найти область определения функции двух переменных $z = f(x, y)$. **Изобразить эту область на координатной плоскости и заштриховать.**

2. Проверить, удовлетворяет ли функция двух переменных $z = f(x, y)$ **указанному дифференциальному уравнению (задачи а) и б))**

3. Найти наименьшее и наибольшее значения функции $z = f(x; y)$ **в замкнутой области D, заданной системой неравенств. Сделать чертеж области D.**

ВАРИАНТ 1

. 1) $z = \sqrt{x^2 + y^2 - 4}.$

2) а) $z = y \cdot \sin(x^2 - y^2)$, $\frac{1}{x} \cdot \frac{\partial z}{\partial x} + \frac{1}{y} \cdot \frac{\partial z}{\partial y} = \frac{z}{y^2}$;

б) $z = x \cdot \sin\frac{y}{x} + y \ln\frac{y}{x}$, $y\frac{\partial^2 z}{\partial y^2} + x\frac{\partial^2 z}{\partial x \partial y} = 0.$

3) $z = x^2 + y^2 - 9xy + 27$; $0 \le x \le 3,\ 0 \le y \le 3.$

ВАРИАНТ 2

1) $z = \dfrac{1}{\sqrt{(x-2)(y+1)}}$.

2) а) $z = y \cdot e^{x^2 - y^2}$, $\dfrac{1}{x} \cdot \dfrac{\partial z}{\partial x} + \dfrac{1}{y} \cdot \dfrac{\partial z}{\partial y} = \dfrac{z}{y^2}$;

б) $z = x \cdot e^{\frac{y}{x}} + y \cdot \ln \dfrac{y}{x}$, $x^2 \cdot \dfrac{\partial^2 z}{\partial x^2} + 2xy \dfrac{\partial^2 z}{\partial x \partial y} + y^2 \cdot \dfrac{\partial^2 z}{\partial y^2} = 0.$

3) $z = x^2 + 2y^2 + 1;$ $x \geq 0, \quad y \geq 0, \quad x + y \leq 3.$

ВАРИАНТ 3

1) $z = \sqrt{1 - x^2 - y^2}$.

2) а) $z = x \cdot \sin(x^2 - y^2),$ $x^2 \cdot \dfrac{\partial z}{\partial y} + xy \dfrac{\partial z}{\partial x} = zy;$

б) $z = \cos^2(x + y) + \ln(x - y),$ $\dfrac{\partial^2 z}{\partial x^2} = \dfrac{\partial^2 z}{\partial y^2}.$

3) $z = 3 - 2x^2 - xy - y^2;$ $x \leq 1, \quad y \geq 0, \quad y \leq x.$

ВАРИАНТ 4

1) $z = \sqrt{\dfrac{y-x}{x^2+y^2-1}}$.

2) а) $z = tg^3(2x-3y)$, $3\dfrac{\partial z}{\partial x} + 2\dfrac{\partial z}{\partial y} = 0$;

б) $z = e^{x+y^2}$, $2\dfrac{\partial^2 z}{\partial x^2} + 2y\dfrac{\partial z}{\partial y} = \dfrac{\partial^2 z}{\partial y^2}$.

3) $z = x^2 + 3y^2 + x - y$; $x \ge 1,\ y \ge -1,\ x + y \le 1$.

ВАРИАНТ 5

283. 1) $z = \sqrt{1 - \dfrac{x^2}{4} - \dfrac{y^2}{9}}$.

2) а) $z = e^{\frac{x}{y^2}}$, $2x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 0$;

б) $z = \sin^2(x-y) + \ln(x+y)$, $\dfrac{\partial^2 z}{\partial x^2} = \dfrac{\partial^2 z}{\partial y^2}$.

3) $z = x^2 + 2xy + 2y^2$; $-1 \le x \le 1,\ 0 \le y \le 2$.

ВАРИАНТ 6

1) $z = \frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}}$.

2) а) $z = x \cdot e^{y^2 - x^2}$, $x^2 \cdot \frac{\partial z}{\partial y} + xy\frac{\partial z}{\partial x} = zy;$

б) $z = \sin(x + y^2)$, $\frac{\partial^2 z}{\partial y^2} = 2\frac{\partial z}{\partial x} + 4y^2\frac{\partial^2 z}{\partial x^2}.$

3) $z = 5x^2 - 3xy + y^2 + 4;$ $x \geq -1,\ y \geq -1,\ x + y \leq 1.$

ВАРИАНТ 7

1) $z = \sqrt{xy}$.

2) а) $z = \ln(x^2 + xy + y^2)$, $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2$;

б) $z = \sin(x + \ln y)$, $\frac{\partial z}{\partial x}\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial z}{\partial y}\frac{\partial^2 z}{\partial x^2}.$

3) $z = 10 + 2xy - x^2;$ $0 \leq y \leq 4 - x^2.$

ВАРИАНТ 8

1) $z = \sqrt{x \ln y}$.

2) а) $z = y \ln(x^2 - y^2)$, $\frac{1}{x} \cdot \frac{\partial z}{\partial x} + \frac{1}{y} \cdot \frac{\partial z}{\partial y} = \frac{z}{y^2}$;

б) $z = \frac{\sin(x - y)}{y}$, $\frac{\partial}{\partial y}\left(y^2 \frac{\partial z}{\partial y} \right) = y^2 \frac{\partial^2 z}{\partial x^2}$.

3) $z = x^2 + 2xy - y^2 + 4x$; $x \le 0,\ y \le 0,\ x + y + 2 \ge 0$.

ВАРИАНТ 9

1) $z = \ln(x^2 + y^2 - 4)$.

2) а) $z = x \cdot e^{y^2 - x^2}$, $\frac{1}{y} \cdot \frac{\partial z}{\partial y} + \frac{1}{x} \cdot \frac{\partial z}{\partial x} = \frac{z}{x^2}$;

б) $z = x \ln \frac{y}{x} + y \cos \frac{y}{x}$, $x \frac{\partial^2 z}{\partial x^2} + y \frac{\partial^2 z}{\partial x \partial y} = 0$.

3) $z = x^2 + xy - 2$; $4x^2 - 4 \le y \le 0$.

ВАРИАНТ 10

1) $z = \dfrac{\sqrt{2xy}}{\sqrt{x^2 + y^2 - 4}}$.

2) а) $z = e^{\frac{y}{x}} + \dfrac{x}{y}$, $x \cdot \dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 0$;

б) $z = e^x(x\cos y - y\sin y)$, $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$.

3) $z = x^2 + xy;$ $-1 \le x \le 1,\ 0 \le y \le 3.$

ВАРИАНТ 11

1) $z = \ln\dfrac{x+y}{y-3}$.

2) а) $z = e^{\frac{y}{x}} + \dfrac{x}{y}$, $x \cdot \dfrac{\partial z}{\partial x} + y \cdot \dfrac{\partial z}{\partial y} = 0$;

б) $z = \ln(e^x + e^y)$, $\dfrac{\partial^2 z}{\partial x^2} \cdot \dfrac{\partial^2 z}{\partial y^2} - \left(\dfrac{\partial^2 z}{\partial x \partial y}\right)^2 = 0$.

3) $z = 2x^2 - 4xy + 5y^2 - 8x + 6;$ $0 \le x \le 4;\ \ 0 \le y \le 4$.

ВАРИАНТ 12

1) $z = \sqrt{9 - x^2 - y^2}$.

2) а) $z = \sin\dfrac{x}{y}$, $x \cdot \dfrac{\partial z}{\partial x} + y \cdot \dfrac{\partial z}{\partial y} = 0$;

б) $z = \dfrac{\ell^{x+y}}{x}$, $x\dfrac{\partial^2 z}{\partial x^2} + 2\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{\partial^2 z}{\partial y^2}$.

3) $z = 2x^2 - 4xy + 5y^2 - 8x + 6;$ $0 \le x \le 4;\quad 0 \le y \le 4$.

ВАРИАНТ 13

1) $z = \ln(xy)$.

2) а) $z = \ln(e^x + e^y)$, $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = 1$;

б) $z = \ln(x - 2y) + \cos(x + 2y)$, $4\dfrac{\partial^2 z}{\partial x^2} = \dfrac{\partial^2 z}{\partial y^2}$.

3) $z = 5x^2 + 8xy + 5y^2 - 18x - 18y;$ $0 \le x \le 2;\quad 0 \le y \le 3$.

ВАРИАНТ 14

1) $z = \sqrt{(x-2)(y+4)}$.

2) а) $z = \dfrac{x^2 - y^2}{x^2 + y^2}$, $x \cdot \dfrac{\partial z}{\partial x} + y \cdot \dfrac{\partial z}{\partial y} = 0$;

б) $z = x\ln(x+y) + y\sin(x+y)$, $\dfrac{\partial^2 z}{\partial y^2} - 2\dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial^2 z}{\partial x^2} = 0$.

3) $z = 2xy - 3x^2 - 3y^2 + 4x + 4y;$ $0 \le x \le 3;\quad 0 \le y \le 2$.

ВАРИАНТ 15

1) $z = \sqrt{\dfrac{4+x}{y-5}}$.

2) а) $z = ctg\,\dfrac{x}{y}$, $x \cdot \dfrac{\partial z}{\partial x} + y \cdot \dfrac{\partial z}{\partial y} = 0$;

б) $z = \dfrac{\ln(x+y)}{x}$, $\dfrac{\partial}{\partial x}\left(x^2 \dfrac{\partial z}{\partial x}\right) \cdot = x^2 \dfrac{\partial^2 z}{\partial y^2}$.

3) $z = x^3 + y^3 - 3xy;$ $0 \le x \le 2;\quad 0 \le y \le 3$.

ВАРИАНТ 16

1) $z = \sqrt{\frac{x^2}{9} + \frac{y^2}{4} - 1}$.

2) а) $z = tg\frac{y}{x}$, $x \cdot \frac{\partial z}{\partial x} + y \cdot \frac{\partial z}{\partial y} = 0$;

б) $z = \frac{x}{x^2 - 9y^2}$, $\frac{\partial^2 z}{\partial y^2} = 9\frac{\partial^2 z}{\partial x^2}$.

3) $z = \frac{x^2}{2} - xy$ $y \geq \frac{x^2}{3}; \quad y \leq 3$.

ВАРИАНТ 17

1) $z = \sqrt{\frac{x+2}{y-1}}$.

2) а) $z = \frac{x^3 - y^3}{x^3 - 4y^3}$, $x \cdot \frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0;$

б) $z = \frac{\sin(x+y)}{x}$, $\frac{\partial}{\partial x}\left(x^2 \frac{\partial z}{\partial x}\right) \cdot = x^2 \frac{\partial^2 z}{\partial y^2}$.

3) $z = x^2 - xy + y^2 - 4x;$ $x \geq 0;\ y \geq 0;\ 2x + 3y \leq 12$.

ВАРИАНТ 18

1) $z = \sqrt{\dfrac{\ln x}{y}}.$

2) а) $z = \ln(x^2 + y^2)$, $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y} = 0$;

б) $z = \dfrac{\cos(x+y)}{x}$, $\dfrac{\partial}{\partial x}\left(x^2\dfrac{\partial z}{\partial x}\right) = x^2\dfrac{\partial^2 z}{\partial y^2}$.

3) $z = x^2 + y^2 - xy + x + y;$ $x \le 0;\ y \le 2;\ x + y \ge -3$.

ВАРИАНТ 19

1) $z = \ln(16 - x^2 - y^2)$.

2) а) $z = tg(x^2 + y^2)$, $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y} = 0$;

б) $z = x\sin(x+y) + y\cdot e^{x+y}$, $\dfrac{\partial^2 z}{\partial x^2} - 2\dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial^2 z}{\partial y^2} = 0$.

3) $z = x^3 + 8y^3 - 6xy + 1;$ $0 \le x \le 2;\quad -1 \le y \le 1$.

ВАРИАНТ 20

1) $z = \ln(x(y^2 - 1)).$

2) а) $z = \dfrac{y}{x^2 - y^2}$, $\dfrac{1}{x} \cdot \dfrac{\partial z}{\partial x} + \dfrac{1}{y} \cdot \dfrac{\partial z}{\partial y} = \dfrac{z}{y^2}$;

б) $z = \dfrac{\ln(x - y)}{x}$, $\dfrac{\partial}{\partial x}\left(x^2 \dfrac{\partial z}{\partial x} \right) \cdot = x^2 \dfrac{\partial^2 z}{\partial y^2}$.

3) $z = x^3 + y^3 - 9xy - 25;$ $0 \le x \le 3; \quad 0 \le y \le 3$.

ВАРИАНТ 21

. 1) $z = \sqrt{x^2 + y^2 - 4}.$

2) а) $z = y \cdot \sin(x^2 - y^2),$ $\dfrac{1}{x} \cdot \dfrac{\partial z}{\partial x} + \dfrac{1}{y} \cdot \dfrac{\partial z}{\partial y} = \dfrac{z}{y^2}$;

б) $z = x \cdot \sin\dfrac{y}{x} + y \ln\dfrac{y}{x}$, $y\dfrac{\partial^2 z}{\partial y^2} + x\dfrac{\partial^2 z}{\partial x \partial y} = 0.$

3) $z = x^2 + y^2 - 9xy + 27$; $0 \le x \le 3, \ 0 \le y \le 3.$

ВАРИАНТ 22

1) $z = \dfrac{1}{\sqrt{(x-2)(y+1)}}$.

2) а) $z = y \cdot e^{x^2 - y^2}$, $\dfrac{1}{x} \cdot \dfrac{\partial z}{\partial x} + \dfrac{1}{y} \cdot \dfrac{\partial z}{\partial y} = \dfrac{z}{y^2}$;

б) $z = x \cdot e^{\frac{y}{x}} + y \cdot \ln \dfrac{y}{x}$, $x^2 \cdot \dfrac{\partial^2 z}{\partial x^2} + 2xy \dfrac{\partial^2 z}{\partial x \partial y} + y^2 \cdot \dfrac{\partial^2 z}{\partial y^2} = 0.$

3) $z = x^2 + 2y^2 + 1;$ $x \geq 0, \quad y \geq 0, \quad x + y \leq 3.$

ВАРИАНТ 23

1) $z = \sqrt{1 - x^2 - y^2}$.

2) а) $z = x \cdot \sin(x^2 - y^2),$ $x^2 \cdot \dfrac{\partial z}{\partial y} + xy \dfrac{\partial z}{\partial x} = zy;$

б) $z = \cos^2(x + y) + \ln(x - y),$ $\dfrac{\partial^2 z}{\partial x^2} = \dfrac{\partial^2 z}{\partial y^2}.$

3) $z = 3 - 2x^2 - xy - y^2;$ $x \leq 1, \quad y \geq 0, \quad y \leq x.$

ВАРИАНТ 24

1) $z = \sqrt{\dfrac{y-x}{x^2+y^2-1}}$.

2) а) $z = tg^3(2x-3y)$, $3\dfrac{\partial z}{\partial x} + 2\dfrac{\partial z}{\partial y} = 0$;

б) $z = e^{x+y^2}$, $2\dfrac{\partial^2 z}{\partial x^2} + 2y\dfrac{\partial z}{\partial y} = \dfrac{\partial^2 z}{\partial y^2}$.

3) $z = x^2 + 3y^2 + x - y$; $x \ge 1,\ y \ge -1,\ x + y \le 1$.

ВАРИАНТ 25

283. 1) $z = \sqrt{1 - \dfrac{x^2}{4} - \dfrac{y^2}{9}}$.

2) а) $z = e^{\frac{x}{y^2}}$, $2x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 0$;

б) $z = \sin^2(x-y) + \ln(x+y)$, $\dfrac{\partial^2 z}{\partial x^2} = \dfrac{\partial^2 z}{\partial y^2}$.

3) $z = x^2 + 2xy + 2y^2$; $-1 \le x \le 1,\ 0 \le y \le 2$.

ВАРИАНТ 26

1) $z = \frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}}$.

2) а) $z = x \cdot e^{y^2 - x^2}$, $x^2 \cdot \frac{\partial z}{\partial y} + xy\frac{\partial z}{\partial x} = zy;$

б) $z = \sin(x + y^2)$, $\frac{\partial^2 z}{\partial y^2} = 2\frac{\partial z}{\partial x} + 4y^2\frac{\partial^2 z}{\partial x^2}.$

3) $z = 5x^2 - 3xy + y^2 + 4;$ $x \geq -1,\ y \geq -1,\ x + y \leq 1.$

ВАРИАНТ 27

1) $z = \sqrt{xy}$.

2) а) $z = \ln(x^2 + xy + y^2)$, $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2$;

б) $z = \sin(x + \ln y)$, $\frac{\partial z}{\partial x}\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial z}{\partial y}\frac{\partial^2 z}{\partial x^2}.$

3) $z = 10 + 2xy - x^2;$ $0 \leq y \leq 4 - x^2.$

ВАРИАНТ 28

1) $z = \sqrt{x \ln y}$.

2) а) $z = y \ln(x^2 - y^2)$, $\dfrac{1}{x} \cdot \dfrac{\partial z}{\partial x} + \dfrac{1}{y} \cdot \dfrac{\partial z}{\partial y} = \dfrac{z}{y^2}$;

б) $z = \dfrac{\sin(x - y)}{y}$, $\dfrac{\partial}{\partial y}\left(y^2 \dfrac{\partial z}{\partial y} \right) = y^2 \dfrac{\partial^2 z}{\partial x^2}$.

3) $z = x^2 + 2xy - y^2 + 4x$; $x \le 0,\ y \le 0,\ x + y + 2 \ge 0$.

ВАРИАНТ 29

1) $z = \ln(x^2 + y^2 - 4)$.

2) а) $z = x \cdot e^{y^2 - x^2}$, $\dfrac{1}{y} \cdot \dfrac{\partial z}{\partial y} + \dfrac{1}{x} \cdot \dfrac{\partial z}{\partial x} = \dfrac{z}{x^2}$;

б) $z = x \ln \dfrac{y}{x} + y \cos \dfrac{y}{x}$, $x \dfrac{\partial^2 z}{\partial x^2} + y \dfrac{\partial^2 z}{\partial x \partial y} = 0$.

3) $z = x^2 + xy - 2$; $4x^2 - 4 \le y \le 0$.

ВАРИАНТ 30

1) $z = \dfrac{\sqrt{2xy}}{\sqrt{x^2 + y^2 - 4}}$.

2) а) $z = e^{\frac{y}{x}} + \dfrac{x}{y}$, $x \cdot \dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 0$;

б) $z = e^x(x\cos y - y\sin y)$, $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$.

3) $z = x^2 + xy;$ $-1 \le x \le 1,\ 0 \le y \le 3.$