

4. Найти площадь фигуры, ограниченной данными линиями.

1. $x^2 + y^2 - 2y = 0,$

$x^2 + y^2 - 4y = 0, y = x/\sqrt{3},$

$y = \sqrt{3}x.$

2. $x^2 - 4x + y^2 = 0,$

$x^2 - 8x + y^2 = 0, y = 0,$

$y = x/\sqrt{3}.$

3. $x^2 + y^2 - 6y = 0,$

$x^2 + y^2 - 8y = 0, x = \sqrt{3}y,$

$y = \sqrt{3}x.$

4. $x^2 - 2x + y^2 = 0,$

$x^2 - 4x + y^2 = 0, y = 0,$

$y = x.$

5. $x^2 + y^2 - 8y = 0,$

$x^2 + y^2 - 10y = 0, y = x/\sqrt{3},$

$y = \sqrt{3}x.$

6. $x^2 - 4x + y^2 = 0,$

$x^2 - 8x + y^2 = 0, y = 0,$

$y = x.$

7. $x^2 + y^2 - 4y = 0,$

$x^2 + y^2 - 6y = 0, x = 0,$

$y = x.$

8. $x^2 - 2x + y^2 = 0,$

$x^2 - 10x + y^2 = 0, y = 0,$

$y = \sqrt{3}x.$

9. $x^2 + y^2 - 6y = 0,$

$x^2 + y^2 - 10y = 0, x = 0,$

$y = x.$

10. $x^2 - 2x + y^2 = 0,$

$x^2 - 4x + y^2 = 0, x = \sqrt{3}y,$

$y = \sqrt{3}x.$

11. $x^2 + y^2 - 2y = 0,$

$x^2 + y^2 - 4y = 0, x = 0,$

$y = \sqrt{3}x.$

12. $x^2 - 2x + y^2 = 0,$

$x^2 - 6x + y^2 = 0, y = x/\sqrt{3},$

$y = \sqrt{3}x.$

13. $x^2 + y^2 - 4y = 0,$

$x^2 + y^2 - 6y = 0, x = 0,$

$y = \sqrt{3}x.$

14. $x^2 - 2x + y^2 = 0,$

$x^2 - 8x + y^2 = 0, y = x/\sqrt{3},$

$y = \sqrt{3}x.$

15. $x^2 + y^2 - 2y = 0,$

$x^2 + y^2 - 6y = 0, x = 0,$

$y = x/\sqrt{3}.$

16. $x^2 - 2x + y^2 = 0,$

$x^2 - 4x + y^2 = 0, y = 0,$

$y = x/\sqrt{3}.$

В декартовой системе координат вычислить пределы интегрирования в двойном интеграле.

- 17. $x^2+y^2-2y=0,$
 $x^2+y^2-10y=0, y=x/\sqrt{3},$
 $y=\sqrt{3}x.$
- 18. $x^2-2x+y^2=0,$
 $x^2-6x+y^2=0, y=0,$
 $y=x/\sqrt{3}.$
- 19. $x^2+y^2-4y=0,$
 $x^2+y^2-10y=0, y=x/\sqrt{3},$
 $y=\sqrt{3}x.$
- 20. $x^2-2x+y^2=0,$
 $x^2-6x+y^2=0, y=0,$
 $y=x.$
- 21. $x^2+y^2-2y=0,$
 $x^2+y^2-4y=0, x=0,$
 $y=x.$
- 22. $x^2-2x+y^2=0,$
 $x^2-4x+y^2=0, y=0,$
 $y=\sqrt{3}x.$
- 23. $x^2+y^2-6y=0,$
 $x^2+y^2-8y=0, x=0,$
 $y=x.$
- 24. $x^2-4x+y^2=0,$
 $x^2-8x+y^2=0, y=0,$
 $y=\sqrt{3}x.$
- 25. $x^2+y^2-4y=0,$
 $x^2+y^2-8y=0, x=0,$
 $y=x.$
- 26. $x^2-4x+y^2=0,$
 $x^2-8x+y^2=0, y=x/\sqrt{3},$
 $y=\sqrt{3}x.$
- 27. $x^2+y^2-4y=0,$
 $x^2+y^2-8y=0, x=0,$
 $y=\sqrt{3}x.$
- 28. $x^2-4x+y^2=0,$
 $x^2-6x+y^2=0, y=\sqrt{3}x,$
 $y=x/\sqrt{3}.$
- 29. $x^2+y^2-2y=0,$
 $x^2+y^2-10y=0, x=0,$
 $y=x/\sqrt{3}.$
- 30. $x^2-6x+y^2=0,$
 $x^2-10x+y^2=0, y=\sqrt{3}x,$
 $y=x/\sqrt{3}.$

5. Пластинка D задана ограничивающими ее кривыми, $\mu(x,y)$ - поверхностная плотность. Найти массу пластины.

- 1. $x=1, y=0, y^2=4x (y \geq 0);$ 2. $x^2+y^2=1, x^2+y^2=4, x=0,$
 $y=0 (x \geq 0, y \geq 0);$
 $\mu(x,y) = 7x^2+y.$ $\mu(x,y) = (x+y)/(x^2+y^2).$

3. $x=1, y=0, y^2=4x$
 $(y \geq 0); \mu(x,y) = 7x^2/2 + 5y.$

11. $x=1, y=0, y^2=x (y \geq 0);$
 $\mu(x,y) = 3x + 6y^2.$

4. $x^2+y^2=9, x^2+y^2=16$
 $(x \geq 0, y \geq 0); \mu(x,y) =$
 $(2x+5y)/(x^2+y^2).$

12. $x^2+y^2=9, x^2+y^2=25$
 $(x \leq 0, y \geq 0); \mu(x,y) =$
 $(2y-x)/(x^2+y^2).$

5. $x=2, y=0, y^2=2x$
 $(y \geq 0); \mu(x,y) = 7x^2/8 + 2y.$

13. $x=2, y=0, y^2=x/2 (y \geq 0);$
 $\mu(x,y) = 2x + 3y^2.$

6. $x^2+y^2=1, x^2+y^2=16$
 $(x \geq 0, y \geq 0); \mu(x,y) =$
 $(x+y)/(x^2+y^2).$

14. $x^2+y^2=4, x^2+y^2=16$
 $(x \leq 0, y \geq 0); \mu(x,y) =$
 $(2y-5x)/(x^2+y^2).$

7. $x=2, y=0, y^2=x/4$
 $(y \geq 0); \mu(x,y) = 7x^2/2 + 6y.$

15. $x=1/2, y=0, y^2=3x$
 $(y \geq 0); \mu(x,y) = 7x + 3y^2.$

8. $x^2+y^2=4, x^2+y^2=25$
 $(x \geq 0, y \leq 0); \mu(x,y) =$
 $(2x-3y)/(x^2+y^2).$

16. $x^2+y^2=9, x^2+y^2=16$
 $(x \leq 0, y \geq 0); \mu(x,y) =$
 $(2y-5x)/(x^2+y^2).$

9. $x=1, y=0, y^2=4x$
 $(y \geq 0); \mu(x,y) = x + 3y^2.$

17. $x=1, y=0, y^2=8x$
 $(y \geq 0); \mu(x,y) = 7x^2 + 2y.$

10. $x^2+y^2=1, x^2+y^2=9$
 $(x \geq 0, y \leq 0); \mu(x,y) =$
 $(x-y)/(x^2+y^2).$

18. $x^2+y^2=1, x^2+y^2=16$
 $(x \geq 0, y \geq 0); \mu(x,y) =$
 $(x+3y)/(x^2+y^2).$

19. $x=2, y=0, y^2=2x$
($y \geq 0$); $\mu(x, y) =$
 $= 7x^2/4 + y/2.$

20. $x^2+y^2=1, x^2+y^2=4$
($x \geq 0, y \geq 0$); $\mu(x, y) =$
 $= (x+2y)/(x^2+y^2).$

21. $x=2, y=0, y^2=2x$
($y \geq 0$); $\mu(x, y) = 7x^2/4 + y.$

22. $x^2+y^2=1, x^2+y^2=9$
($x \geq 0, y \leq 0$); $\mu(x, y) =$
 $= (2x-y)/(x^2+y^2).$

23. $x=2, y=0, y^2=x/2$
($y \geq 0$); $\mu(x, y) = 7x^2/2 + 8y.$

24. $x^2+y^2=1, x^2+y^2=25$
($x \geq 0, y \leq 0$); $\mu(x, y) =$
 $= (x-4y)/(x^2+y^2).$

25. $x=1, y=0, y^2=4x$ ($y \geq 0$)
 $\mu(x, y) = 6x + 3y^2.$

26. $x^2+y^2=4, x^2+y^2=16$
($x \geq 0, y \leq 0$); $\mu(x, y) =$
 $= (3x-y)/(x^2+y^2).$

27. $x=2, y=0, y^2=x/2$
($y \geq 0$); $\mu(x, y) = 4x + 6y^2.$

28. $x^2+y^2=4, x^2+y^2=9$
($x \leq 0, y \geq 0$); $\mu(x, y) =$
 $= (y-4x)/(x^2+y^2).$

29. $x=1/2, y=0, y^2=2x$
($y \geq 0$); $\mu(x, y) = 4x + 9y^2.$

30. $x^2+y^2=4, x^2+y^2=9$
($x \leq 0, y \geq 0$); $\mu(x, y) =$
 $= (y-2x)/(x^2+y^2).$

6. Пластина D задана неравенствами, $\mu(x, y)$ — поверхностная плотность. Найти массу пластины.

1. $x^2+y^2/4 \leq 1, x \geq 0;$

2. $x^2/9 + y^2/25 \leq 1, y \geq 0;$

7) Тело V задано ограничивающими его поверхностями, μ - плотность. Найти массу тела.

1. $y = 16\sqrt{2x}$, $y = \sqrt{2x}$, $z = 0$,
 $x + z = 2$; $\mu = y^2/13$.
2. $y = 5\sqrt{x}$, $y = 5x/3$, $z = 0$,
 $z = 5 + 5\sqrt{x}/3$; $\mu = 4y/(3x)$.
3. $x^2 + y^2 = 2$, $y = \sqrt{x}$, $y = 0$,
 $z = 0$, $z = 15x$; $\mu = 16x^2y$.
4. $x + y = 2$, $y = \sqrt{x}$, $z = 0$,
 $z = 12y$; $\mu = 20xy^2$.
5. $x = 20\sqrt{2y}$; $x = 5\sqrt{2y}$, $z = 0$,
 $x + y = 1/2$; $\mu = 8y/(3x^3)$.
6. $x = 5\sqrt{y}/2$, $x = 5y/6$, $z = 0$,
 $z = 3 + \sqrt{y}$; $\mu = 14x/5$.
7. $x^2 + y^2 = 2$, $x = \sqrt{y}$, $x = 0$,
 $z = 0$, $z = 30y$; $\mu = 20y^2$.
8. $x + y = 2$, $x = \sqrt{y}$, $z = 0$,
 $z = 12x/5$; $\mu = 15xy$.
9. $y = 17\sqrt{2x}$, $y = 2\sqrt{2x}$, $z = 0$,
 $x + z = 1/2$; $\mu = 143x^4$.
10. $y = 5\sqrt{x}/3$, $y = 5x/9$, $z = 0$,
 $z = 5(3 + \sqrt{x})/9$; $\mu = 84y/25$.
11. $x^2 + y^2 = 8$, $y = \sqrt{2x}$, $y = 0$,
 $z = 0$, $z = 15x/11$; $\mu = 1,1x^2y$.
12. $x + y = 4$, $y = \sqrt{2x}$, $z = 3y$,
 $z = 0$; $\mu = 35xy^2/4$.
13. $x = 5\sqrt{y}/6$, $x = 5y/18$, $z = 0$,
 $z = 5(3 + \sqrt{y})/18$; $\mu = 7y/3$.
14. $x = 19\sqrt{2y}$, $x = 4\sqrt{2y}$, $z = 0$,
 $x + z = 2$; $\mu = 5x$.
15. $x^2 + y^2 = 8$, $x = \sqrt{2y}$, $x = 0$,
 $z = 5y$, $z = 0$; $\mu = 3xy^2$.
16. $x + y = 4$, $x = \sqrt{2y}$, $z = 3x/5$,
 $z = 0$; $\mu = 5y$.
17. $y = 6\sqrt{3x}$, $y = \sqrt{3x}$, $z = 0$,
 $x + z = 3$; $\mu = 7x$.
18. $y = 5\sqrt{x}/6$, $y = 5x/18$, $z = 0$,
 $z = 5(3 + \sqrt{x})/18$; $\mu = 7x^2$.
19. $x^2 + y^2 = 18$, $y = \sqrt{3x}$, $y = 0$,
 $z = 0$, $z = 2x/9$; $\mu = 4y$.
20. $x + y = 6$, $y = \sqrt{3x}$, $z = 0$,
 $z = 4y$; $\mu = y/(9x^2)$.
21. $x = 7\sqrt{3y}$, $x = 2\sqrt{3y}$, $z = 0$,
 $z + y = 3$; $\mu = 4xy/27$.
22. $x = 5\sqrt{y}/3$, $x = 5y/9$, $z = 0$,
 $z = 5(3 + \sqrt{y})/9$; $\mu = x/(4y)$.

23. $x^2 + y^2 = 18, x = \sqrt{3}y, z = 0,$
 $z = 0, z = 10y/3; \mu = xy/4.$
24. $x + y = 6, x = \sqrt{3}y, z = 4x,$
 $z = 0; \mu = 35x^2y/27.$
25. $y = \sqrt{15}x, y = \sqrt{15}x, z = 0,$
 $z = \sqrt{15}(1 + \sqrt{x}); \mu = 13x.$
26. $x^2 + y^2 = 50, y = \sqrt{5}x, y = 0,$
 $z = 0, z = 3x/17; \mu = 7y^2.$
27. $x + y = 8, y = \sqrt{4}x, z = 3y,$
 $z = 0; \mu = 35xy/29.$
28. $x = 16\sqrt{2}y, x = \sqrt{2}y, z = 0,$
 $y + z = 2; \mu = xy.$
29. $x = 15\sqrt{y}, x = 15y, z = 0,$
 $z = 15(1 + \sqrt{y}); \mu = 7y^2.$
30. $x^2 + y^2 = 50, x = \sqrt{5}y, x = 0,$
 $z = 0, z = 4y/5; \mu = xy^2.$

В. Найти объем тела V , заданного ограничивающими его поверхностями.

1. $x^2 + y^2 = 2y, z = 5/4 - x,$
 $z = 0.$
2. $x^2 + y^2 = y, x^2 + y^2 = 4y,$
 $z = 0, z = \sqrt{x^2 + y^2}.$
3. $x^2 + y^2 = 2\sqrt{x}x, z = 0,$
 $z = x^2 + y^2 - 4.$
4. $x^2 + y^2 = -4x, z = 8 - y,$
 $z = 0.$
5. $x^2 + y^2 = 2x, x^2 + y^2 = 5x,$
 $z = \sqrt{x^2 + y^2}, z = 0, y = 0 (y \leq 0).$
6. $x^2 + y^2 = 3\sqrt{2}y, z = 0,$
 $z = x^2 + y^2 - 9.$
7. $x^2 + y^2 = 2y, z = 2 - x,$
 $z = 0.$
8. $x^2 + y^2 = y, x^2 + y^2 = 3y,$
 $z = 0, z = x^2 + y^2.$
9. $x^2 + y^2 = -2\sqrt{2}y, z = x^2 + y^2 - 4,$
 $z = 0.$
10. $x^2 + y^2 = 4x, z = 3 - y,$
 $z = 0.$
11. $x^2 + y^2 = 7x, x^2 + y^2 = 10x,$
 $z = \sqrt{x^2 + y^2}, y = 0 (y \leq 0).$
12. $x^2 + y^2 = 2\sqrt{2}y, z = 0,$
 $z = x^2 + y^2 - 4.$
13. $x^2 + y^2 = 2y, z = 7 - x,$
 $z = 0.$
14. $x^2 + y^2 = 5y, x^2 + y^2 = y,$
 $z = x^2 + y^2, z = 0.$
15. $x^2 + y^2 = 3\sqrt{2}y, z = 0,$
 $z = x^2 + y^2 - 9.$
16. $x^2 + y^2 = -2\sqrt{2}y, z = 0,$
 $z = x^2 + y^2 - 4.$

17. $x^2 + y^2 = 4x$, $z = 12 - y$,
 $z = 0$.
18. $x^2 + y^2 = 7x$, $x^2 + y^2 = 9x$,
 $z = x^2 + y^2$, $z = 0$.
19. $x^2 + y^2 = 4\sqrt{2}$, $z = 0$,
 $z = x^2 + y^2 - 16$.
20. $x^2 + y^2 = 4y$, $z = 4 - x$,
 $z = 0$.
21. $x^2 + y^2 = 4y$, $x^2 + y^2 = 7y$,
 $y = 0$ ($y > 0$), $z = \sqrt{x^2 + y^2}$, $z = 0$.
22. $x^2 + y^2 = 4\sqrt{2}y$, $z = 0$,
 $z = x^2 + y^2 - 16$.
23. $x^2 + y^2 = -4x$, $z = 17/4 - y$,
 $z = 0$.
24. $x^2 + y^2 = 9x$, $x^2 + y^2 = 12x$,
 $z = \sqrt{x^2 + y^2}$, $z = 0$.
25. $x^2 + y^2 = -2\sqrt{2}x$, $z = 0$,
 $z = x^2 + y^2 - 4$.
26. $x^2 + y^2 = 4y$, $z = 6 - x$,
 $z = 0$.
27. $x^2 + y^2 = 10x$, $x^2 + y^2 = 13x$,
 $z = \sqrt{x^2 + y^2}$, $z = 0$ ($y > 0$).
28. $x^2 + y^2 = 2\sqrt{2}y$, $z = 0$,
 $z = x^2 + y^2 - 4$.
29. $x^2 + y^2 = 6$, $z = 21/4 - y$,
 $z = 0$.
30. $x^2 + y^2 = 5y$, $x^2 + y^2 = 8y$,
 $z = \sqrt{x^2 + y^2}$, $z = 0$.