

I. Найти общий интеграл дифференциального уравнения:

- 1) а) $3(x^2y + y) dy + \sqrt{2 + y^2} dx = 0$;
б) $xy' = 4\sqrt{2x^2 + y^2} + y$;
в) $y' = (x + 6y - 7)/(8x - y - 7)$.
- 2) а) $y' = (3y - 2x + 1)/(3x + 3)$;
б) $\sqrt{3 + y^2} + \sqrt{1 - x^2} y \cdot y' = 0$;
в) $xy' = 4\sqrt{x^2 + y^2} + y$.
- 3) а) $xy' = (3y^3 + 14yx^2)/(2y^2 + 7x^2)$;
б) $y' = (x + y - 4)/(x - 2)$;
в) $y(1 + \ln y) + xy' = 0$.
- 4) а) $\sqrt{1 - x^2} y' + xy^2 + x = 0$;
б) $xy' = 2\sqrt{3x^2 + y^2} + y$;
в) $y' = y/(2x + 2y - 2)$.
- 5) а) $y' = (2x + y - 3)/(4x - 4)$;
б) $xy' = (3y^3 + 12yx^2)/(2y^2 + 6x^2)$;
в) $y \ln y + xy' = 0$.
- 6) а) $\sqrt{5 + y^2} + y'y\sqrt{1 - x^2} = 0$;
б) $y' = (x + 4y - 5)/(6x - y - 5)$;
в) $xy' = 3\sqrt{2x^2 + y^2} + y$.

- 7) a) $xy' = (3y^3 + 10yx^2)/(2y^2 + 5x^2)$;
 б) $\sqrt{4 - x^2} y' + xy^2 + x = 0$;
 в) $y' = (3x + 2y - 1)/(x + 1)$.
- 8) a) $y' = (y - 2x + 3)/(x - 1)$;
 б) $xy' = 3\sqrt{x^2 + y^2} + y$;
 в) $y'y\sqrt{(1 - x^2)/(1 - y^2)} + 1 = 0$.
- 9) а) $\sqrt{3 + y^2} dx - y dy = x^2 y dy$;
 б) $y' = (4y - 8)/(3x + 2y - 7)$;
 в) $xy' = (3y^3 + 8yx^2)/(2y^2 + 4x^2)$.
- 10) а) $xy' = \sqrt{2x^2 + y^2} + y$;
 б) $x\sqrt{1 + y^2} + yy'\sqrt{1 + x^2} = 0$;
 в) $y' = (y + 2)/(2x + y - 4)$.
- 11) а) $y' = (6y - 6)/(5x + 4y - 9)$;
 б) $20x dx - 3y dy = 3x^2 y dy - 5xy^2 dx$;
 в) $xy' = (3y^3 + 6yx^2)/(2y^2 + 3x^2)$.
- 12) а) $xy' = 2\sqrt{x^2 + y^2} + y$;
 б) $2x + 2xy^2 + \sqrt{2 - x^2} y' = 0$;
 в) $y' = (2x + y - 1)/(2x - 2)$.
- 13) а) $y' = (x + 5y - 6)/(7x - y - 6)$;
 б) $xy' = (3y^3 + 4xy^2)/(2y^2 + 2x^2)$;
 в) $2x dx - y dy = yx^2 dy - xy^2 dx$.
- 14) а) $(1 + e^x)yy' = e^x$;
 б) $y' = (2x + y - 3)/(2x - 2)$;
 в) $xy' = \sqrt{x^2 + y^2} + y$.
- 15) а) $xy' = (3y^3 + 2yx^2)/(2y^2 + x^2)$;
 б) $\sqrt{5 + y^2} dx + 4(x^2 y + y) dy = 0$;
 в) $y' = (x + y + 2)/(x + 1)$.
- 16) а) $y' = (5y + 5)/(4x + 3y - 1)$;
 б) $y' = (x^2 + 2xy - 5y^2)/(2x^2 - 6xy)$;
 в) $yy' = e^x/(1 + e^x)$.
- 17) а) $x dx - y dy = yx^2 dy - xy^2 dx$;
 б) $y'(x - 1) = x + 2y - 3$;
 в) $3x^2 y' = y^2 + 10yx - 10x^2$.
- 18) а) $y'(x^2 - 6xy) = x^2 + xy - 5y^2$;
 б) $6x dx - 2y dy = 2yx^2 dy - 3xy^2 dx$;
 в) $y'(5x - y - 4) = x + 3y - 4$.

- 19) a) $y' = (2x + 3y - 5)/(5x - 5)$;
 б) $4x^2y' = y^2 + 10yx + 5x^2$;
 в) $(1 + e^x)y' = ye^x$.
- 20) a) $6x dx - y dy = yx^2 dy - 3xy^2 dx$;
 б) $y' = (x + 8y - 9)/(10x - y - 9)$;
 в) $y' = (x^2 + xy - 3y^2)/(x^2 - 4xy)$.
- 21) a) $y' = y^2/x^2 + 8y/x + 12$;
 б) $(e^x + 8) dy - ye^x dx = 0$;
 в) $y'(4x - y - 3) = (x + 2y - 3)$.
- 22) a) $y' = (x + 3y + 4)/(3x - 6)$;
 б) $y' = (x^2 + 3xy - y^2)/(3x^2 - 2xy)$;
 в) $x\sqrt{4 + y^2} dx + y\sqrt{1 + x^2} dy = 0$.
- 23) a) $2x dx - 2y dy = x^2y dy - 2xy^2 dx$;
 б) $y' = (2x + y - 3)/(x - 1)$;
 в) $2y' = y^2/x^2 + y/x + 8$.
- 24) a) $y(4 + e^x) dy - e^x dx = 0$;
 б) $y' = (x^2 + 2xy - y^2)/(2x^2 - 2xy)$;
 в) $y' = (2y - 2)/(x + y - 2)$.
- 25) a) $y' = y^2/x^2 + 6y/x + 6$;
 б) $y'(2x - 2) = x + y - 2$;
 в) $x\sqrt{5 + y^2} dx + y\sqrt{4 + x^2} dy = 0$.
- 26) a) $y' = (x - 2y + 3)/(-2x - 2)$;
 б) $y' = (x^2 + xy - y^2)/(x^2 - 2xy)$;
 в) $6x dx - 6y dy = 3x^2y dy - 2xy^2 dx$.
- 27) a) $(e^{2x} + 5) dy + ye^{2x} dx = 0$;
 б) $y' = (8y + 3)/(2x + y - 1)$;
 в) $3y' = y^2/x^2 + 8y/x + 4$.
- 28) a) $y' = (x + 2y)/(2x - y)$;
 б) $y' = (x + 7y - 8)/(9x - y - 8)$;
 в) $x\sqrt{3 + y^2} dx + y\sqrt{2 + x^2} dy = 0$.
- 29) a) $6x dx - 6y dy = 2x^2y dy - 3xy^2 dx$;
 б) $y' = (x + y)/(x - y)$;
 в) $y'(3x - y - 2) = (x + y - 2)$.
- 30) a) $y' = y^2/x^2 + 4y/x + 2$;
 б) $y' = (3y - x - 4)/(3x + 3)$;
 в) $\sqrt{4 + y^2} dx - y dy = x^2y dy$.

II. Решить задачу Коши. Нарисовать интегральную кривую.

- 1) а) $y' - y \cos x = \sin 2x, \quad y(0) = -1;$
б) $xy' + y = xy^2, \quad y(1) = 1.$
- 2) а) $2(y' + xy) = (x - 1)e^2y^2, \quad y(0) = 2;$
б) $y' - y/x = \ln x/x, \quad y(1) = 1.$
- 3) а) $y' - y \cos x = -\sin 2x, \quad y(0) = 3;$
б) $y' + y = xy^2, \quad y(0) = 1.$
- 4) а) $y' - y = xy^2, \quad y(0) = 1;$
б) $y' - y/x = -2/x^2, \quad y(1) = 1.$
- 5) а) $y' + xy = (x - 1)e^xy^2, \quad y(0) = 1;$
б) $y' - 3x^2y = x^2(1 + x^2)/3, \quad y(0) = 0.$
- 6) а) $y' - 4xy = -4x^3, \quad y(0) = -0,5;$
б) $8xy' - 12y = -(5x^2 + 3)y^3, \quad y(1) = \sqrt{2}.$
- 7) а) $2y' + 3y \cos x = 12 \cos x e^xy^{-1}, \quad y(0) = 2;$
б) $y' - 2y/(x + 1) = (x + 1)^3, \quad y(0) = 0,5.$
- 8) а) $y' + 2xy = xe^{-x^2} \sin x, \quad y(0) = 1;$
б) $y' + 2xy = 2x^3y^3, \quad y(0) = \sqrt{2}.$
- 9) а) $y' + xy = -x^3, \quad y(0) = 3;$
б) $y' - y = 2xy^2, \quad y(0) = 0,5.$
- 10) а) $2y' + 3y \cos x = 3e^x \cos x y^{-1}, \quad y(0) = 1;$
б) $y' + 2xy = -2x^3, \quad y(1) = e^{-1}.$
- 11) а) $y' + y(1 - 2x)/x^2 = 1, \quad y(1) = 1;$
б) $2xy' - 3y = -(5x^2 + 3)y^3, \quad y(1) = 1/\sqrt{2}.$
- 12) а) $y' + y/x = 3x, \quad y(1) = 1;$
б) $y' + 4x^3y = 4y^2e^{4x}(1 - x^3), \quad y(0) = -1.$
- 13) а) $3y' + 2xy = 2xy^{-2}e^{-2x^2}, \quad y(0) = -1;$
б) $y' - y/x = -12/x^3, \quad y(1) = 4.$
- 14) а) $y' + y/x = (x + 1)e^x/x, \quad y(1) = e;$
б) $3xy' + 5y = (4x - 5)y^4, \quad y(1) = 1.$
- 15) а) $3(xy' + y) = xy^2, \quad y(1) = 3;$
б) $y' + 2xy/(1 + x^2) = 2x^2/(1 + x^2), \quad y(0) = \frac{2}{3}.$
- 16) а) $y' + y/x = \sin x, \quad y(\pi) = 1/\pi;$
б) $2xy' - 3y = -(20x^2 + 12)y^3, \quad y(1) = 1/2\sqrt{2}.$
- 17) а) $xy' + y = y^2 \ln x, \quad y(1) = 1;$
б) $y' - y/(x + 1) = e^x(x + 1), \quad y(0) = 1.$
- 18) а) $y' + y \operatorname{tg} x = \cos^2 x, \quad y(\pi/4) = 0,5;$
б) $4y' + x^3y = (x^3 + 8)e^{-2xy^2}, \quad y(0) = 1.$

- 19) a) $2(y' + y) = xy^2, \quad y(0) = 2;$
 б) $y' - y \operatorname{ctg} x = 2x \sin x, \quad y(\pi/2) = 0.$
- 20) a) $y' - y/x = x^2, \quad y(1) = 0;$
 б) $2y' - 3y \cos x = -e^{-2x}(2 + 3 \cos x)y^{-1}, \quad y(0) = 1.$
- 21) a) $2(xy' + y) = y^2 \ln x, \quad y(1) = 2;$
 б) $y' + y \cos x = \frac{1}{2} \sin 2x, \quad y(0) = 0.$
- 22) a) $y' - y/(x+2) = x^2 + 2x, \quad y(-1) = 1, 5;$
 б) $y' + 2y \operatorname{ctg} x = y^2 \cos x, \quad y(1) = 2.$
- 23) a) $y' - y \operatorname{tg} x = -(2/3)y^4 \sin x, \quad y(0) = 1;$
 б) $y' - y/x = x \sin x, \quad y(\pi/2) = 1.$
- 24) a) $y' + xy = (1+x)e^{-x}y^2, \quad y(0) = 1;$
 б) $y' + y/(2x) = x^2, \quad y(1) = 1.$
- 25) a) $2(xy' + y) = xy^2, \quad y(1) = 2;$
 б) $y' - (2x-5)y/x^2 = 5, \quad y(2) = 4.$
- 26) a) $xy' - y = -y^2(\ln x + 2) \ln x, \quad y(1) = 1;$
 б) $y' - y/x = -2 \ln x/x, \quad y(1) = 1.$
- 27) a) $y' + 2y/x = x^3, \quad y(1) = -5/6;$
 б) $3(xy' + y) = y^2 \ln x, \quad y(1) = 3.$
- 28) a) $2y' + y \cos x = y^{-1} \cos x(1 + \sin x), \quad y(0) = 1;$
 б) $y' - 2xy/(1+x^2) = 1+x^2, \quad y(1) = 3.$
- 29) a) $y' + 3y/x = 2/x^3, \quad y(1) = 1;$
 б) $2(y' + xy) = (1+x)e^{-x}y^2, \quad y(0) = 2.$
- 30) a) $y' + xy/(2(1-x^2)) = x/2, \quad y(0) = \frac{2}{3};$
 б) $y' + 4x^3y = 4(x+1)e^{-4x}y^2, \quad y(0) = 1.$

III. Найти общее решение диф. уравнения

- а) Найдите общее решение дифференциального уравнения методом понижения порядка;
- б) Найдите решение задачи Коши методом неопределённых коэффициентов;
- в) Найдите общее решение методом Лагранжа.

Замечание. В пункте б) нарисовать интегральную кривую.

1. а) $x^3 y'' + x^2 y' = 1$;

б) $y'' - 2y' + y = -12 \cos 2x - 9 \sin 2x$,

$$y(0) = -2, \quad y'(0) = 0;$$

в) $y'' - y = \frac{e^x}{e^x + 1}$.

2.а) $2yy'' = y'^2$;

б) $y'' - 6y' + 9y = 9x^2 - 39x + 65, \quad y(0) = -1, \quad y'(0) = 1$;

в) $y'' + 4y = \frac{1}{\cos 2x}$.

3. а) $y'' + y' \operatorname{tg} x = \sin 2x$;

б) $y'' + 2y' + 2y = 2x^2 + 8x + 6, \quad y(0) = 1, \quad y'(0) = 4$;

в) $y'' - 4y' + 5y = \frac{e^{2x}}{\cos x}$.

4. а) $y'' \operatorname{tg} y = 2y'^2$;

б) $y'' - 6y' + 25y = 9 \sin 4x - 24 \cos 4x$,

$$y(0) = 2, \quad y'(0) = -2;$$

в) $y'' + 9y = \frac{1}{\sin 3x}$.

5. a) $xy'' - y' = x^2e^x$;
b) $y'' - 14y' + 53y = 53x^3 - 42x^2 + 59x - 14$,

$y(0) = 0, \quad y'(0) = 7$;

c) $y'' + 2y' + 2y = \frac{e^{-x}}{\cos x}$.

6. a) $(y'')^2 = y'$;

b) $y'' + 16y = e^x(\cos 4x - 8 \sin 4x)$,

$y(0) = 0, \quad y'(0) = 5$;

c) $y'' - 2y' + 2y = \frac{e^x}{\sin^2 x}$.

7. a) $y''x \ln x = 2y'$;

b) $y'' - 4y' + 20y = 16xe^{2x}, \quad y(0) = 1, \quad y'(0) = 2$;

c) $y'' + 2y' + y = \frac{e^{-x}}{x}$.

8. a) $y'' = y' + y'^2$;

b) $y'' - 12y' + 36y = 32 \cos 2x + 24 \sin 2x$,

$y(0) = 2, \quad y'(0) = 4$;

c) $y'' + 4y = \operatorname{tg} 2x$.

9. a) $xy'' = y'$;

b) $y'' + y = x^3 - 4x^2 + 7x - 10, \quad y(0) = 2, \quad y'(0) = 3$;

c) $y'' - 4y' + 4y = \frac{e^{2x}}{x^3}$.

10. a) $yy'' - 2y'^2 = 0$;

b) $y'' - y = (14 - 16x)e^{-x}, \quad y(0) = 0, \quad y'(0) = -1$;

c) $y'' + y = \frac{2}{\sin^2 x}$.

11. a) $xy'' + y' = \ln x$;

b) $y'' + 8y' + 16y = 16x^2 - 16x + 66$,

$$y(0) = 3, \quad y'(0) = 0;$$

$$c) \quad y'' - y' = e^{2x} \cos(e^x).$$

$$12. \text{ a) } y'' + \frac{2}{1-y} y'^2 = 0;$$

$$b) \quad y'' + 10y' + 34y = -9e^{-5x}, \quad y(0) = 0, \quad y'(0) = 6;$$

$$c) \quad y'' + 9y = \frac{1}{\cos 3x}.$$

$$13. \text{ a) } xy'' = y' + x^2$$

$$b) \quad y'' + 16y = (34x + 13)e^{-x}, \quad y(0) = -1, \quad y'(0) = 5;$$

$$c) \quad y'' - y = \frac{2e^x}{e^x - 1}.$$

$$14. \text{ a) } y''(1+y) = 5y'^2;$$

$$b) \quad y'' + 25y = e^x(\cos 5x - 10 \sin 5x),$$

$$y(0) = 3, \quad y'(0) = -4;$$

$$c) \quad y'' + 2y' + 2y = e^{-x} \operatorname{ctg} x.$$

$$15. \text{ a) } y''' + y'' \operatorname{tg} x = \sec x;$$

$$b) \quad y'' - 10y' + 25y = 2e^{5x}, \quad y(0) = 1, \quad y'(0) = 0;$$

$$c) \quad y'' - 2y' + 2y = \frac{e^x}{\sin x}.$$

$$16. \text{ a) } 1 + y'^2 = yy'';$$

$$b) \quad y'' + y' - 12y = (16x + 26)e^{4x}, \quad y(0) = 3, \quad y'(0) = 5;$$

$$c) \quad y'' + y = \operatorname{tg} x.$$

$$17. \text{ a) } y'' - 2y' \operatorname{ctg} x = \sin^3 x;$$

$$b) \quad y'' - 2y' + 5y = 5x^2 + 6x - 12, \quad y(0) = 0, \quad y'(0) = 2;$$

$$c) \quad y'' - y' = \frac{1}{1 + e^x}.$$

$$18. \text{ a) } yy'' - y'(1 + y') = 0;$$

b) $y'' - 3y' + 2y = e^{3x}(3 - 4x)$, $y(0) = 0$, $y'(0) = 0$;
c) $y'' - 2y' + y = \frac{e^x}{x^2}$.

19. a) $x(y'' + 1) + y' = 0$;
b) $y'' + 8y' + 16y = 16x^3 + 24x^2 - 10x + 8$,
 $y(0) = 1$, $y'(0) = 3$
c) $y'' + 4y = \operatorname{ctg} 2x$.

20. a) $y'' = \frac{y'}{\sqrt{y}}$;
b) $y'' + y = \sin 2x$, $y(0) = 0$, $y'(0) = 0$;
c) $y'' + 4y' + 4y = \frac{e^{-2x}}{x^3}$.

21. a) $y'' - 2y' \operatorname{ctg} x = \sin^3 x$;
b) $y'' - 8y' = 16 + 48x^2 - 128x^3$, $y(0) = -1$, $y'(0) = 14$;
c) $y'' - y' = e^{2x} \sin(e^x)$.

22. a) $y'' = 1 + y'^2$;
b) $y'' - y = 2(1 - x)$, $y(0) = 0$, $y'(0) = 1$;
c) $y'' + y = \operatorname{tg}^2 x$.

23. a) $y'''x \ln x = y''$;
b) $y'' + 3y' = (40x + 58)e^{2x}$, $y(0) = 0$, $y'(0) = -2$;
c) $y'' + 2y' + y = 3e^{-x}\sqrt{x+1}$.

24. a) $y'' + 2yy'^3 = 0$;
b) $y'' - 9y' + 18y = 26 \cos x - 8 \sin x$,
 $y(0) = 0$, $y'(0) = 2$;
c) $y'' + 2y' + 5y = \frac{e^{-x}}{\sin 2x}$.

25. a) $(1 + x^2)y'' = 2xy'$;

b) $y'' + 8y' = 18x + 60x^2 - 32x^3$, $y(0) = 5$, $y'(0) = -16$

c) $y'' - y' = e^{2x}\sqrt{1 - e^{2x}}$.

26. a) $yy'' = y^2y' + y'^2$;

b) $y'' - 3y' + 2y = -\sin x - 7\cos x$, $y(0) = 2$, $y'(0) = 7$;

c) $y'' + y = -\operatorname{ctg}^2 x$.

27. a) $y'' \operatorname{ctg} x + y' = 2$;

b) $y'' + 2y' = 6x^2 + 2x - 2$, $y(0) = 2$, $y'(0) = 2$;

c) $y'' + 4y = \frac{1}{\sin 2x}$.

28. a) $y'' + yy'^3 = 0$;

b) $y'' + 16y = 32e^{4x}$, $y(0) = 2$, $y'(0) = 0$;

c) $y'' - 6y + 9y = 36\sqrt{x}e^{3x}$.

29. a) $2xy''y' = y'^2 - 4$;

b) $y'' + 5y' + 6y = 52\sin 2x$, $y(0) = -2$, $y'(0) = -5$;

c) $y'' - 3y' + 2y = 1 + \frac{1}{1 + e^x}$.

30. a) $y'' = -\frac{1}{2y^3}$;

b) $y'' - 4y = 8e^{2x}$, $y(0) = 1$, $y'(0) = -8$;

c) $y'' + 4y' + 4y = e^{-2x} \ln x$.