

Условия задач

1. Вычислить интеграл по области D , ограниченной указанными линиями.
2. Вычислить интеграл по области D , заданной системой неравенств:
 - изобразить область в декартовой системе координат;
 - вычислить интеграл, переходя к полярным координатам.
3. Найти объем тела, заданного ограничивающими его поверхностями.
4. Найти объем тела, заданного ограничивающими его поверхностями.
5. Найти объем тела, заданного неравенствами.
6. Найти площадь части поверхности S , проектирующейся на область D .
7. Найти заряд пластинки, ограниченной кривыми, если плотность заряда в точке задана функцией $\mu(x, y)$.
8. Найти массу кривой L , если плотность кривой в каждой ее точке равна ординате этой точки.
9. Найти работу силы \vec{F} при перемещении материальной точки вдоль линии L от точки M до точки N .
10. Вычислить интеграл по замкнутому контуру C в положительном направлении по формуле Грина.
11. Даны векторное поле \vec{F} и плоскость $Ax + By + Cz + D = 0$ (p), которая совместно с координатными плоскостями образует пирамиду T . Пусть σ — основание пирамиды, принадлежащее плоскости p ; λ — контур, ограничивающий σ ; \vec{n} — нормаль к σ , направленная вне пирамиды T . Требуется вычислить:
 - поток векторного поля \vec{F} через поверхность σ в направлении нормали \vec{n} ;
 - циркуляцию векторного поля \vec{F} по замкнутому контуру λ непосредственно и применив теорему Стокса к контуру λ и ограниченной им поверхности σ с нормалью \vec{n} ;
 - поток векторного поля \vec{F} через полную поверхность пирамиды T в направлении внешней нормали к ее поверхности непосредственно и применив теорему Гаусса — Остроградского. Сделать чертеж.

ВАРИАНТ 1

1.

$$\iint_D (12x^2y^2 + 16x^3y^3) dx dy$$
$$D : x = 1; y = x^2; y = -\sqrt{x}$$

2.

$$\iint_D (x + y) dx dy$$
$$D : \begin{cases} 0 \leq y \leq \sqrt{3} \cdot x; \\ 2x \leq x^2 + y^2 \leq 4x \end{cases}$$

3.

$$x^2 + y^2 = 18; y = \sqrt{3}x;$$
$$z = \frac{25}{11}x; y = 0; z = 0$$

4.

$$z = \sqrt{4 - x^2 - y^2}; z = \sqrt{\frac{x^2 + y^2}{255}}$$

5.

$$16 \leq x^2 + y^2 + z^2 \leq 100; y \leq 0;$$
$$0 \leq z \leq \sqrt{\frac{x^2 + y^2}{24}}; y \leq -\frac{x}{\sqrt{3}}$$

6.

$$S : z = \sqrt{4 - x^2 - y^2};$$
$$D : \{x^2 + y^2 \leq 1, z = 0\}$$

7.

$$2 \leq \frac{x^2}{5} + \frac{y^2}{3} \leq 5; 0 \leq y \leq \frac{3\sqrt{5}}{5}x;$$
$$\mu = \frac{32}{63}xy$$

8. L — отрезок прямой $y = 2 - \frac{1}{2}x$, заключенный между точками $A(0, 2)$ и $B(4, 0)$.

9.

$$\vec{F} = (x^2 - 2y)\vec{i} + (y^2 - 2x)\vec{j};$$

L — отрезок прямой MN , где $M(-4, 0)$, $N(0, 2)$.

10.

$$\int_C (y^2 - y) dx + (2xy + y) dy;$$
$$C : \{x = \sqrt{9 - y^2}, x = 0\}$$

11.

$$\vec{F} = (x + z)\vec{i} + y\vec{j} + \vec{k}, \quad p : x + y + z - 2 = 0$$

ВАРИАНТ 2

1.

$$\iint_D y^2 \cdot \sin \frac{xy}{2} dx dy$$

$$D : x = 0; y = \sqrt{\pi}; y = \frac{x}{2}$$

2.

$$\iint_D \sqrt{1 - x^2 - y^2} dx dy$$

$$D : \begin{cases} x^2 + y^2 \leq x; \\ x^2 + y^2 \leq y \end{cases}$$

3.

$$y = 2\sqrt{x}; x + y = 3; \\ z = 3x; z = 0$$

4.

$$z = \frac{15}{2}\sqrt{x^2 + y^2}; z = \frac{17}{2} - x^2 - y^2$$

5.

$$25 \leq x^2 + y^2 + z^2 \leq 100; y \geq 0; \\ z \leq -\sqrt{\frac{x^2 + y^2}{99}}; y \geq -x\sqrt{3}$$

6.

$$S : z = 2 - x^2 - y^2; \\ D : \{x^2 + y^2 \leq 1, z = 0\}$$

7.

$$1 \leq \frac{x^2}{3} + 3y^2 \leq 15; x \geq 0; y \geq \frac{1}{3}x; \\ \mu = \frac{x}{y}$$

8. L — контур прямоугольника с вершинами $A(0,1)$, $B(4,1)$, $C(4,2)$ и $D(0,2)$.

9.

$$\vec{F} = (x^2 - 2y)\vec{i} + (y^2 - 2x)\vec{j};$$

$$L : 2 - \frac{x^2}{8} = y,$$

$$M(-4,0), N(0,2).$$

10.

$$\int_C (x+y)^2 dx + (x^2 + y^2) dy;$$

$$C : \{x = \sqrt{16 - y^2}, x = 0\}$$

11.

$$\vec{F} = (x+y)\vec{i} + z\vec{j} + 2\vec{k}, \quad p : 2x - y + 2z - 2 = 0$$

ВАРИАНТ 3

1.

$$\iint_D (36x^2y^2 - 96x^3y^3) dx dy$$
$$D : x = 1; y = \sqrt[3]{x}; y = -x^3$$

2.

$$\iint_D \frac{1}{x^2} \cdot e^{\frac{y}{x}} dx dy$$
$$D : \begin{cases} x \leq y \leq \sqrt{3} \cdot x; \\ 1 \leq x^2 + y^2 \leq 4 \end{cases}$$

3.

$$y = \sqrt{3x}; x + y = 6;$$
$$z = 4y; z = 0$$

4.

$$z = \sqrt{25 - x^2 - y^2}; z = \sqrt{\frac{x^2 + y^2}{99}}$$

5.

$$16 \leq x^2 + y^2 + z^2 \leq 81; y \leq 0;$$
$$z \geq \sqrt{\frac{x^2 + y^2}{99}}; y \leq -x\sqrt{3}$$

6.

$$S : z = x^2 - y^2;$$
$$D : \{x^2 + y^2 \leq 4, z = 0\}$$

7.

$$1 \leq \frac{x^2}{3} + \frac{y^2}{1} \leq 9; -x \leq y \leq 0;$$
$$\mu = \frac{20}{363}xy^2$$

8. L — дуга параболы $y^2 = 2px$, отсеченная параболой $x^2 = 2py$, ($p > 0$).

9.

$$\vec{F} = (x + y)\vec{i} + 2x\vec{j};$$
$$L : x^2 + y^2 = 4 \quad (y \geq 0),$$
$$M(2, 0), N(-2, 0).$$

10.

$$\int_C y dx - x^2 dy;$$
$$C : \{y = x^2 + 1, y = 2\}$$

11.

$$\vec{F} = (y - x + z)\vec{i} - \vec{j}, \quad p : 2x + y + 2z - 4 = 0$$

ВАРИАНТ 4

1.

$$\iint_D y^2 \cdot e^{-\frac{xy}{4}} dx dy$$

$$D : x = 0; y = 2; y = x$$

2.

$$\iint_D \frac{1}{\sqrt{4-x^2-y^2}} dx dy$$

$$D : \begin{cases} y \geq x; y \geq -x; \\ x^2 + y^2 \leq 2y \end{cases}$$

3.

$$y = \sqrt{3x}; x^2 + y^2 = 18;$$

$$y = 0; z = \frac{5}{11}x; z = 0$$

4.

$$z = \sqrt{64 - x^2 - y^2}; z = 1;$$

$$x^2 + y^2 = 60 \text{ (внутри цилиндра)}$$

5.

$$64 \leq x^2 + y^2 + z^2 \leq 144;$$

$$z \geq -\sqrt{\frac{x^2 + y^2}{63}}; 0 \leq y \leq \frac{x}{\sqrt{3}}$$

6.

$$S : 2x + 3y + z = 6;$$

$$D : \left\{ \frac{x^2}{4} + \frac{y^2}{9} \leq 1, z = 0 \right\}$$

7.

$$9 \leq x^2 + \frac{y^2}{3} \leq 100; 0 \leq y \leq 3x;$$

$$\mu = \frac{y}{x^2}$$

8. L — контур треугольника с вершинами $A(-1, 2)$, $B(1, 2)$ и $C(0, 1)$.

9.

$$\vec{F} = y^2 \vec{i} + x^2 \vec{j};$$

$$L : \begin{cases} x = a \cos t \\ y = b \sin t \end{cases}, y \geq 0,$$

$$M(-a, 0), N(a, 0).$$

10.

$$\int_C e^y [(x - 2 \sin x) dx - (1 - 2 \cos x) dy];$$

$$C : \{y = x^2 - 1, y = 0\}$$

11.

$$\vec{F} = x \vec{i} + (y - z) \vec{j} + 2 \vec{k}, \quad p : -x + 2y + 2z - 4 = 0$$

ВАРИАНТ 5

1.

$$\iint_D (9x^2y^2 + 48x^3y^3) dx dy$$

$$D : x = 1; y = \sqrt{x}; y = -x^2$$

2.

$$\iint_D \operatorname{arctg} \frac{y}{x} dx dy$$

$$D : \begin{cases} 0 \leq y \leq \frac{x}{\sqrt{3}}; \\ 2x \leq x^2 + y^2 \leq 4x \end{cases}$$

3.

$$x + y = 3; y^2 = 4x;$$

$$z = -y; z = 0 \quad (z \geq 0)$$

4.

$$z = \frac{21}{2} \sqrt{x^2 + y^2};$$

$$z = \frac{23}{2} - x^2 - y^2$$

5.

$$64 \leq x^2 + y^2 + z^2 \leq 196;$$

$$z \leq \sqrt{\frac{x^2 + y^2}{3}}; \frac{x}{\sqrt{3}} \leq y \leq 0$$

6.

$$S : x^2 + y^2 = z^2 \quad (z \geq 0);$$

$$D : \{x^2 + y^2 \leq 2x, z = 0\}$$

7.

$$1 \leq \frac{x^2}{3} + \frac{y^2}{4} \leq 4; x \geq 0; y \geq 2x;$$

$$\mu = \frac{40}{31} x^2 y$$

8.

$$L - \text{часть параболы } y = 2\sqrt{x}, \quad 0 \leq x \leq 1.$$

9.

$$\vec{F} = (x + y) \vec{i} + (x - y) \vec{j};$$

$$L : y = x^2,$$

$$M(-1, 1), N(1, 1).$$

10.

$$\int_C (2xy - y) dx + (x^2 + x) dy;$$

$$C : \{x^2 + y^2 = 9\}$$

11.

$$\vec{F} = (2x + 3y - 3z) \vec{j}, \quad p : 2x - 3y + 2z - 6 = 0$$

ВАРИАНТ 6

1.

$$\iint_D y^2 \cdot \cos \frac{xy}{2} dx dy$$

$$D : x = 0; y = \sqrt{\frac{\pi}{2}}; y = \frac{x}{2}$$

2.

$$\iint_D \sqrt{x^2 + y^2} dx dy$$

$$D : \begin{cases} x \geq 0; y \geq 0; \\ x^2 + y^2 \leq x + y \end{cases}$$

3.

$$\begin{aligned} x &= 3\sqrt{3y}; x = 2\sqrt{3y}; \\ z &= 0; z + y = 3 \end{aligned}$$

4.

$$\begin{aligned} z &= 3\sqrt{x^2 + y^2}; \\ z &= 10 - x^2 - y^2 \end{aligned}$$

5.

$$\begin{aligned} 64 &\leq x^2 + y^2 + z^2 \leq 196; \\ -\sqrt{\frac{x^2 + y^2}{3}} &\leq z \leq \sqrt{\frac{x^2 + y^2}{15}} \end{aligned}$$

6.

$$\begin{aligned} S &: x^2 + y^2 + z^2 = 25 \quad (y \geq 0); \\ D &: \{x^2 + z^2 \leq 16, y = 0\} \end{aligned}$$

7.

$$\begin{aligned} 1 &\leq \frac{x^2}{9} + \frac{y^2}{3} \leq 400; y \geq \frac{x}{3}; x \geq 0; \\ \mu &= \frac{x}{y^2} \end{aligned}$$

8.

$$L : y = \frac{x^2}{4} - \frac{\ln x}{2}, \quad 1 \leq x \leq 2.$$

9.

$$\begin{aligned} \vec{F} &= x\vec{j}; \\ L &: x^2 + y^2 = a^2, x \geq 0, \\ M &(0, -a), N(0, a). \end{aligned}$$

10.

$$\begin{aligned} &\int_C (y^2 + 2 \ln x) dx + xy dy; \\ C &: \{x = y^2 + 1, x = 2\} \end{aligned}$$

11.

$$\vec{F} = x\vec{i} + (z - x)\vec{j} + y\vec{k}, \quad p : 3x + 2y + 3z - 6 = 0$$

ВАРИАНТ 7

1.

$$\iint_D (18x^2y^2 + 32x^3y^3) dx dy$$

$$D : x = 1; y = x^3; y = -\sqrt[3]{x}$$

2.

$$\iint_D (1 - x^2 - y^2) dx dy$$

$$D : \begin{cases} 0 \leq y \leq x\sqrt{3}; \\ x^2 + y^2 \leq 4x \end{cases}$$

3.

$$y = \sqrt{5x}; x^2 + y^2 = 50$$

$$z = \frac{12}{25}x; z = 0 \quad (z \geq 0)$$

4.

$$z = \sqrt{9 - x^2 - y^2}; z = \sqrt{\frac{x^2 + y^2}{80}}$$

5.

$$16 \leq x^2 + y^2 + z^2 \leq 100;$$

$$-\sqrt{\frac{x^2 + y^2}{3}} \leq z \leq \sqrt{\frac{x^2 + y^2}{35}}$$

6.

$$S : z = 1 - x^2 - y^2;$$

$$D : \{x^2 + y^2 \leq 1, z = 0\}$$

7.

$$9 \leq \frac{x^2}{8} + \frac{y^2}{2} \leq 10; y \geq \frac{x}{2}; x \geq 0;$$

$$\mu = \frac{x}{y}$$

8.

$$L : y = x^3, \quad 0 \leq x \leq 2.$$

9.

$$\vec{F} = (x^2 - y^2)\vec{i} + xy\vec{j};$$

$$L : \text{отрезок } MN,$$

$$M(1, 1), N(3, 4).$$

10.

$$\int_C (2y + x^2) dx + (y^2x + y - x) dy;$$

$$C : \{x = y^2 - 1, x = 0\}$$

11.

$$\vec{F} = \vec{i} + (5x + 2y + 3z)\vec{k}, \quad p : x + y + 3z - 3 = 0$$

ВАРИАНТ 8

1.

$$\iint_D 4y^2 \cdot \sin xy \, dx \, dy$$

$$D : x = 0; y = \sqrt{\frac{\pi}{2}}; y = x$$

2.

$$\iint_D \frac{1}{(4 + x^2 + y^2)^2} \, dx \, dy$$

$$D : \begin{cases} x \geq 0; y \geq 0; \\ 2y \leq x^2 + y^2 \leq 4 \end{cases}$$

3.

$$\begin{aligned} x &= 19\sqrt{2y}; x = 4\sqrt{2y}; \\ z &= 0; y + z = 2 \end{aligned}$$

4.

$$\begin{aligned} z &= \sqrt{100 - x^2 - y^2}; z = 6; \\ x^2 + y^2 &= 51 \text{ (внутри цилиндра)} \end{aligned}$$

5.

$$\begin{aligned} 9 &\leq x^2 + y^2 + z^2 \leq 81; y \leq 0; \\ 0 &\leq z \leq \sqrt{\frac{x^2 + y^2}{24}}; y \leq \frac{x}{\sqrt{3}} \end{aligned}$$

6.

$$\begin{aligned} S &: x = z^2 - y^2; \\ D &: \{y^2 + z^2 \leq 4, x = 0\} \end{aligned}$$

7.

$$\begin{aligned} 1 &\leq \frac{x^2}{7} + \frac{y^2}{1} \leq 4; y \geq \frac{\sqrt{7}}{7}x; x \leq 0; \\ \mu &= \frac{15}{31}x^2y \end{aligned}$$

8.

$$L : \begin{cases} x = 5(t - \sin t) \\ y = 5(1 - \cos t) \end{cases}, \quad 0 \leq t \leq \pi$$

9.

$$\begin{aligned} \vec{F} &= y\vec{i} - (y + x^2)\vec{j}; \\ L &: y = 2x - x^2, \\ M(0, 0), N(2, 0). \end{aligned}$$

10.

$$\begin{aligned} \int_C (y - x)^2 \, dx - (x + y)^2 \, dy; \\ C : \{y = \cos x, y = 0, -\pi/2 \leq x \leq \pi/2\} \end{aligned}$$

11.

$$\vec{F} = x\vec{i} + \vec{j} + (y + z)\vec{k}, \quad p : 2x + y + 2z - 4 = 0$$

ВАРИАНТ 9

1.

$$\iint_D (27x^2y^2 + 48x^3y^3) dx dy$$

$$D : x = 1; y = x^2; y = -\sqrt[3]{x}$$

2.

$$\iint_D \frac{y}{x^2} dx dy$$

$$D : \begin{cases} 0 \leq y \leq x; \\ 16 \leq x^2 + y^2 \leq 4\sqrt{2} \cdot x \end{cases}$$

3.

$$x = \frac{5}{3}\sqrt{y}; x = \frac{5}{9}y;$$

$$z = 0; z = \frac{5}{9}(3 + \sqrt{y})$$

4.

$$z = \sqrt{36 - x^2 - y^2};$$

$$z = \sqrt{\frac{x^2 + y^2}{63}}$$

5.

$$49 \leq x^2 + y^2 + z^2 \leq 169; y \geq 0;$$

$$-\sqrt{\frac{x^2 + y^2}{24}} \leq z \leq 0; y \geq \frac{x}{\sqrt{3}}$$

6.

$$S : 2x + y + 3z = 6;$$

$$D : \left\{ \frac{x^2}{4} + \frac{z^2}{9} \leq 1, y = 0 \right\}$$

7.

$$9 \leq \frac{x^2}{2} + \frac{y^2}{6} \leq 25; y \geq x; x \leq 0;$$

$$\mu = \frac{x\sqrt{6}}{y^2}$$

8.

$$L : \begin{cases} x = 10 \cos^3 t \\ y = 10 \sin^3 t \end{cases}, \quad 0 \leq t \leq \frac{\pi}{2}$$

9.

$$\vec{F} = \left(\frac{1}{\sqrt{x}} \right) \vec{i} - \left(\frac{1}{\sqrt{y}} \right) \vec{j};$$

$$L : \begin{cases} x = r \cos t \\ y = r \sin t \end{cases}, \quad x \geq 0, y \geq 0,$$

$$M(r, 0), N(0, r).$$

10.

$$\int_C (x^2 + y^2) dx + (y^2 - x^2) dy;$$

$$C : \left\{ x = -\sqrt{R^2 - y^2}, x = 0 \right\}$$

11.

$$\vec{F} = (z + x) \vec{i} + y \vec{j} + \vec{k}, \quad p : 2x + 2y + z - 4 = 0$$

ВАРИАНТ 10

1.

$$\iint_D y^2 \cdot e^{-\frac{xy}{8}} dx dy$$

$$D : x = 0; y = 2; y = \frac{x}{2}$$

2.

$$\iint_D \frac{1}{\sqrt{3-x^2-y^2}} dx dy$$

$$D : \begin{cases} 0 \leq y \leq x\sqrt{3}; \\ x^2 + y^2 \leq \sqrt{3} \cdot y \end{cases}$$

3.

$$x = \sqrt{2y}; x + y = 4;$$

$$z = 0; z = \frac{3}{5}x$$

4.

$$z = \sqrt{81 - x^2 - y^2}; z = 5;$$

$$x^2 + y^2 = 45 \text{ (внутри цилиндра)}$$

5.

$$9 \leq x^2 + y^2 + z^2 \leq 64; y \leq \frac{x}{\sqrt{3}};$$

$$z \geq \sqrt{\frac{x^2 + y^2}{99}}; y \leq -\frac{x}{\sqrt{3}}$$

6.

$$S : x^2 + y^2 + z^2 = 1 \quad (z \geq 0);$$

$$D : \{x^2 + y^2 \leq x, z = 0\}$$

7.

$$4 \leq \frac{x^2}{3} + \frac{y^2}{9} \leq 100; y \leq -x; y \geq 0;$$

$$\mu = \frac{y}{x^2}$$

8.

$$L : \begin{cases} x = \ln t \\ y = \frac{1}{2}(t + \frac{1}{t}) \end{cases}, 1 \leq t \leq 2$$

9.

$$\vec{F} = 2xy\vec{i} - y^2\vec{j};$$

$$L : \text{отрезок } MN,$$

$$M(0, 0), N(2, 1).$$

10.

$$\int_C (y - x) dx - (x + y) dy;$$

$$C : \left\{ y = b\sqrt{1 - \frac{x^2}{a^2}}, y = 0 \right\}, a > 0, b > 0$$

11.

$$\vec{F} = 2\vec{i} + (y + z)\vec{j} + x\vec{k}, \quad p : 2x + 2y - z - 2 = 0$$

ВАРИАНТ 11

1.

$$\iint_D (18x^2y^2 + 32x^3y^3) dx dy$$

$$D: x = 1; y = \sqrt[3]{x}; y = -x^2$$

2.

$$\iint_D \frac{1}{(4 - x^2 - y^2)^2} dx dy$$

$$D: \begin{cases} x^2 + y^2 \leq 2y; \\ x^2 + y^2 \leq 2x \end{cases}$$

3.

$$x = \sqrt{3y}; x + y = 6;$$

$$z = 0; z = \frac{4}{5}x$$

4.

$$z = \sqrt{100 - x^2 - y^2}; z = 7;$$

$$x^2 + y^2 = 51 \text{ (внутри цилиндра)}$$

5.

$$36 \leq x^2 + y^2 + z^2 \leq 100;$$

$$z \geq -\sqrt{\frac{x^2 + y^2}{63}}; -\frac{x}{\sqrt{3}} \leq y \leq x\sqrt{3}$$

6.

$$S: x^2 + y^2 + z^2 = 4 \quad (x \geq 0);$$

$$D: \{z^2 + y^2 \leq 2z, x = 0\}$$

7.

$$1 \leq \frac{x^2}{4} + \frac{y^2}{3} \leq 2; y \geq \frac{3}{2}x; y \geq 0;$$

$$\mu = xy$$

8.

$$L: \begin{cases} x = e^t(\cos t + \sin t) \\ y = e^t(\cos t - \sin t) \end{cases}, 0 \leq t \leq \frac{\pi}{4}$$

9.

$$\vec{F} = 4xy\vec{i} - x^2\vec{j};$$

$$L: y = \frac{1}{4}x^2,$$

$$M(0, 0), N(2, 1).$$

10.

$$\int_C xy^2 dx - x^2y dy;$$

$$C: \{x = \sqrt{a^2 - y^2}, x = 0\}$$

11.

$$\vec{F} = (z - y + x)\vec{j} - \vec{k}, \quad p: 2x + 2y + z - 4 = 0$$

ВАРИАНТ 12

1.

$$\iint_D y^2 \cdot \cos xy \, dx \, dy$$

$$D : x = 0; y = \sqrt{\pi}; y = x$$

2.

$$\iint_D \frac{x^2}{y^2} \, dx \, dy$$

$$D : 2 \leq x^2 + y^2 \leq -2y$$

3.

$$y = \frac{5}{6}\sqrt{x}; y = \frac{5}{18}x;$$

$$z = 0; z = \frac{5}{18}(3 + \sqrt{x})$$

4.

$$z = 6\sqrt{x^2 + y^2};$$

$$z = 16 - x^2 - y^2$$

5.

$$36 \leq x^2 + y^2 + z^2 \leq 144;$$

$$z \leq \sqrt{\frac{x^2 + y^2}{3}}; x\sqrt{3} \leq y \leq \frac{x}{\sqrt{3}}$$

6.

$$S : x^2 + y^2 + z^2 = 4 \quad (y \geq 0);$$

$$D : \{x^2 + z^2 \leq 2x, y = 0\}$$

7.

$$1 \leq \frac{x^2}{3} + y^2 \leq 4; y \leq 0; y \leq x;$$

$$\mu = x^2y$$

8.

$$L : \begin{cases} x = \cos t + t \sin t \\ y = \sin t - t \cos t \end{cases}, 0 \leq t \leq \frac{\pi}{2}$$

9.

$$\vec{F} = 2xy\vec{i} - x^2\vec{j};$$

$$L : x = 2y^2,$$

$$M(0, 0), N(2, 1).$$

10.

$$\int_C y^2 \, dx - x \, dy;$$

$$C : \{y = \cos x, y = 0, -\pi/2 \leq x \leq \pi/2\}$$

11.

$$\vec{F} = 2\vec{i} + y\vec{j} + (z - x)\vec{k}, \quad p : 2x - y + 2z - 4 = 0$$

ВАРИАНТ 13

1.

$$\iint_D (18x^2y^2 + 32x^3y^3) dx dy$$

$$D : x = 1; y = x^3; y = -\sqrt{x}$$

2.

$$\iint_D \operatorname{arctg} \frac{x}{y} dx dy$$

$$D : \begin{cases} x^2 + y^2 \leq 4y; \\ y \geq 2 \end{cases}$$

3.

$$y = 6\sqrt{3x}; y = \sqrt{3x};$$

$$z = 0; x + z = 3$$

4.

$$z = \sqrt{9 - x^2 - y^2};$$

$$z = \sqrt{\frac{x^2 + y^2}{35}}$$

5.

$$36 \leq x^2 + y^2 + z^2 \leq 144;$$

$$-\sqrt{\frac{x^2 + y^2}{3}} \leq z \leq -\sqrt{\frac{x^2 + y^2}{15}}$$

6.

$$S : y = x^2 - z^2;$$

$$D : \{x^2 + z^2 \leq 9, y = 0\}$$

7.

$$1 \leq \frac{x^2}{4} + \frac{y^2}{9} \leq 4; 0 \leq y \leq \frac{3}{2}x;$$

$$\mu = xy$$

8.

$$L : \begin{cases} x = t^3 + 1 \\ y = t^2 \end{cases}, \quad 0 \leq t \leq 1$$

9.

$$\vec{F} = 2yx\vec{i} - y^2\vec{j};$$

L — ломаная линия, первое звено которой соединяет точки $M(0, 0)$ и $B(2, 0)$, а второе — точки $B(2, 0)$ и $N(2, 1)$.

10.

$$\int_C e^x [(1 + \cos y) dx - (y + \sin y) dy];$$

$$C : \{y = x, x = 2, y = 1\}$$

11.

$$\vec{F} = (2y + 3z - 3x)\vec{k}, \quad p : 2x + 2y - 3z - 6 = 0$$

ВАРИАНТ 14

1.

$$\iint_D 4y^2 \cdot \sin 2xy \, dx \, dy$$

$$D : x = 0; y = \sqrt{2\pi}; y = 2x$$

2.

$$\iint_D \frac{x^2}{y^3} \, dx \, dy$$

$$D : \begin{cases} x^2 + y^2 \geq y; & y \geq x; \\ x^2 + y^2 \leq x + y \end{cases}$$

3.

$$y = 2\sqrt{x}; x + y = 3;$$

$$z = 0; z = 3y \quad (z \geq 0)$$

4.

$$z = \sqrt{64 - x^2 - y^2}; z = 4;$$

$$x^2 + y^2 = 39 \quad (\text{внутри цилиндра})$$

5.

$$9 \leq x^2 + y^2 + z^2 \leq 81;$$

$$-\sqrt{\frac{x^2 + y^2}{3}} \leq z \leq \sqrt{\frac{x^2 + y^2}{35}}$$

6.

$$S : z^2 = y^2 + x^2 \quad (z \geq 0);$$

$$D : \{x \geq 0, y \geq 0, x + y \leq 1\}$$

7.

$$1 \leq \frac{x^2}{1} + \frac{y^2}{3} \leq 9; \quad -x \leq y \leq 0;$$

$$\mu = xy^2$$

8.

$$L : \begin{cases} x = 2(2 \cos t - \cos 2t) \\ y = 2(2 \sin t - \sin 2t) \end{cases}, \quad 0 \leq t \leq \frac{\pi}{3}$$

9.

$$\vec{F} = \cos y \cdot \vec{i} - \sin x \cdot \vec{j};$$

$$L : \text{отрезок } MN,$$

$$M(2, -2), N(-2, 2).$$

10.

$$\int_C \frac{y}{x} \, dx + 2 \ln x \, dy;$$

$$C : \{2x + y = 4, x = 1, y = 0\}$$

11.

$$\vec{F} = z\vec{i} + y\vec{j} + (x - y)\vec{k}, \quad p : 3x + 3y + 2z - 6 = 0$$

ВАРИАНТ 15

1.

$$\iint_D (27x^2y^2 + 48x^3y^3) dx dy$$

$$D : x = 1; y = \sqrt{x}; y = -x^3$$

2.

$$\iint_D \sqrt{4 - x^2 - y^2} dx dy$$

$$D : \begin{cases} x^2 + y^2 \leq 2y; & x \leq 0; \\ x^2 + y^2 \geq -2x \end{cases}$$

3.

$$x + y = 8; y^2 = 4x;$$

$$z = 0; z = 6y \quad (z \geq 0)$$

4.

$$z = \sqrt{16 - x^2 - y^2};$$

$$z = \sqrt{\frac{x^2 + y^2}{15}}$$

5.

$$4 \leq x^2 + y^2 + z^2 \leq 64; y \leq x\sqrt{3};$$

$$0 \leq z \leq \sqrt{\frac{x^2 + y^2}{24}}; y \leq \frac{x}{\sqrt{3}}$$

6.

$$S : z^2 = 2x^2 + 2y^2 \quad (z \geq 0);$$

$$D : \{x^2 + y^2 \leq 2y, z = 0\}$$

7.

$$1 \leq \frac{x^2}{3} + y^2 \leq 4; 0 \leq y \leq x;$$

$$\mu = xy$$

8.

$$L : \rho = 3e^{(3\varphi/4)}, \quad 0 \leq \varphi \leq \frac{\pi}{3}$$

9.

$$\vec{F} = -\frac{y^2}{x^{\frac{5}{3}} + y^{\frac{5}{3}}} \vec{i} + \frac{x^2}{x^{\frac{5}{3}} + y^{\frac{5}{3}}} \vec{j};$$

$$L : \begin{cases} x = R \cos^3 t \\ y = R \sin^3 t \end{cases}, \quad x \geq 0, y \geq 0,$$

$$M(R, 0), N(0, R).$$

10.

$$\int_C (x^2y + x - y) dx + (y^2 + 2x) dy;$$

$$C : \{y = x^2 + 1, y = 2\}$$

11.

$$\vec{F} = (5y + 2z + 3x) \vec{i} + \vec{j}, \quad p : 3x + y + z - 3 = 0$$

ВАРИАНТ 16

1.

$$\iint_D y^2 \cdot e^{-\frac{xy}{2}} dx dy$$

$$D : x = 0; y = \sqrt{2}; y = x$$

2.

$$\iint_D \frac{1}{(x^2 + y^2)^2} dx dy$$

$$D : \begin{cases} x^2 + y^2 \leq 2x; \\ x \geq 1 \end{cases}$$

3.

$$y = \frac{5}{3}\sqrt{x}; y = \frac{5}{9}x;$$

$$z = 0; z = \frac{5}{3}(3 + \sqrt{x})$$

4.

$$z = \frac{3}{2}\sqrt{x^2 + y^2};$$

$$z = \frac{5}{2} - x^2 - y^2$$

5.

$$36 \leq x^2 + y^2 + z^2 \leq 144; y \geq \frac{x}{\sqrt{3}};$$

$$-\sqrt{\frac{x^2 + y^2}{3}} \leq z \leq 0; y \geq x\sqrt{3}$$

6.

$$S : z^2 = 4y^2 + 4x^2 \quad (z \geq 0);$$

$$D : \left\{ \frac{x^2}{4} + \frac{y^2}{9} \leq 1, z = 0 \right\}$$

7.

$$1 \leq \frac{x^2}{4} + \frac{y^2}{25} \leq 4; y \geq \frac{5}{2}x; x \geq 0;$$

$$\mu = \frac{x}{y}$$

8.

$$L : x^2 + y^2 = ay \quad (a > 0)$$

9.

$$\vec{F} = (2a - y)\vec{i} - (a - y)\vec{j};$$

$$L : \begin{cases} x = a(t - \sin t) \\ y = a(1 - \cos t) \end{cases},$$

$$M(0, 0), N(2\pi a, 0).$$

10.

$$\int_C (x + y)^2 dx - (x - y)^2 dy;$$

$$C : \{y = \sin x, y = 0, 0 \leq x \leq \pi\}$$

11.

$$\vec{F} = \vec{i} + (y + x)\vec{j} + z\vec{k}, \quad p : x + y + z - 2 = 0$$

ВАРИАНТ 17

1.

$$\iint_D (4xy + 3x^2y^2) dx dy$$

$$D : x = 1; y = x^2; y = -\sqrt{x}$$

2.

$$\iint_D \frac{1}{x+y} dx dy$$

$$D : 1 \leq x^2 + y^2 \leq x + y$$

3.

$$x = \sqrt{3y}; z = \frac{4}{3}x;$$

$$z = 0; x + y = 6$$

4.

$$z = \sqrt{36 - x^2 - y^2}; z = 2;$$

$$x^2 + y^2 = 27 \text{ (внутри цилиндра)}$$

5.

$$36 \leq x^2 + y^2 + z^2 \leq 121; y \geq 0;$$

$$z \leq -\sqrt{\frac{x^2 + y^2}{99}}; y \geq x\sqrt{3}$$

6.

$$S : y^2 = 2x^2 + 2z^2 \quad (y \geq 0);$$

$$D : \{x \geq 0, z \geq 0, x + 2z \leq 1\}$$

7.

$$1 \leq x^2 + \frac{y^2}{2} \leq 4; y \leq \frac{x\sqrt{6}}{3}; y \leq 0;$$

$$\mu = xy^2$$

8.

$$L : x^2 + y^2 = bx, y \geq 0 \quad (b > 0)$$

9.

$$\vec{F} = 2xy\vec{i} + x^2\vec{j};$$

$$L : y = x^3,$$

$$M(0, 0), N(1, 1).$$

10.

$$\int_C (x^2 - y^2) dx - (x^2 + y^2) dy;$$

$$C : \{y = \sqrt{R^2 - x^2}, y = 0\}$$

11.

$$\vec{F} = y\vec{i} + 2\vec{j} + (z + x)\vec{k}, \quad p : x - 2y - 2z + 2 = 0$$

ВАРИАНТ 18

1.

$$\iint_D y^2 \cdot \cos 2xy \, dx \, dy$$

$$D : x = 0; y = \sqrt{\frac{\pi}{2}}; y = \frac{x}{2}$$

2.

$$\iint_D \frac{y^3}{x^2} \, dx \, dy$$

$$D : \begin{cases} 2x \leq x^2 + y^2 \leq 4x; \\ -\frac{x}{\sqrt{3}} \leq y \leq x \end{cases}$$

3.

$$x = \sqrt{2y}; z = \frac{15}{8}y;$$

$$x^2 + y^2 = 8; x = 0; z = 0$$

4.

$$z = \sqrt{49 - x^2 - y^2}; z = 3;$$

$$x^2 + y^2 = 33 \text{ (внутри цилиндра)}$$

5.

$$4 \leq x^2 + y^2 + z^2 \leq 49; y \leq 0;$$

$$z \geq \sqrt{\frac{x^2 + y^2}{99}}; y \leq x\sqrt{3}$$

6.

$$S : 2y^2 = 3x^2 + 3z^2 \quad (y \geq 0);$$

$$D : \{x^2 + z^2 \leq 2x, y = 0\}$$

7.

$$4 \leq x^2 + \frac{y^2}{3} \leq 9; -3x \leq y \leq 0;$$

$$\mu = \frac{y}{x^2}$$

8.

$$L : \rho = 8 \cos \varphi, \quad 0 \leq \varphi \leq \frac{\pi}{4}$$

9.

$$\vec{F} = (x^2 - y^2) \vec{i} + (x^2 + y^2) \vec{j};$$

$$L : \frac{x^2}{4} + \frac{y^2}{9} = 1, \quad y \geq 0,$$

$$M(3, 0), N(-3, 0).$$

10.

$$\int_C (x + y) \, dx - (x - y) \, dy;$$

$$C : \left\{ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \right\}$$

11.

$$\vec{F} = -\vec{i} + (x - z + y) \vec{k}, \quad p : x + 2y + 2z - 4 = 0$$

ВАРИАНТ 19

1.

$$\iint_D (12xy + 9x^2y^2) dx dy$$

$$D : x = 1; y = \sqrt{x}; y = -x^2$$

2.

$$\iint_D \sqrt{16 - x^2 - y^2} dx dy$$

$$D : \begin{cases} x^2 + y^2 \leq 4; \\ x^2 + y^2 + 4y \geq 0; \\ x \geq 0; y \leq 0 \end{cases}$$

3.

$$y = \sqrt{x}; y = 2\sqrt{x};$$

$$z = 0; z + y = 2$$

4.

$$z = 12\sqrt{x^2 + y^2};$$

$$z = 28 - x^2 - y^2$$

5.

$$16 \leq x^2 + y^2 + z^2 \leq 64; y \geq -\frac{x}{\sqrt{3}};$$

$$z \geq -\sqrt{\frac{x^2 + y^2}{63}}; y \leq -x\sqrt{3}$$

6.

$$S : 3x^2 = 4y^2 + 4z^2 \quad (x \geq 0);$$

$$D : \{y^2 + z^2 \leq 4z, x = 0\}$$

7.

$$4 \leq \frac{x^2}{4} + \frac{y^2}{9} \leq 9; 0 \leq y \leq \frac{3}{2}x;$$

$$\mu = x^2$$

8.

$$L : \rho = 2 \sin \varphi, \quad 0 \leq \varphi \leq \frac{\pi}{6}$$

9.

$$\vec{F} = 2xy\vec{i} - x^2\vec{j};$$

$$L : y = \sin x,$$

$$M(\pi, 0), N(0, 0).$$

10.

$$\int_C xy^2 dx - x^2y dy;$$

$$C : \{x^2 + y^2 = a^2\}$$

11.

$$\vec{F} = (x - z)\vec{i} + 2z\vec{j} + z\vec{k}, \quad p : 2x + 2y - z - 4 = 0$$

ВАРИАНТ 20

1.

$$\iint_D 3y^2 \cdot \sin \frac{xy}{2} dx dy$$

$$D : x = 0; y = \sqrt{\frac{4\pi}{3}}; y = \frac{2}{3}x$$

2.

$$\iint_D \frac{x+y}{x^2+y^2} dx dy$$

$$D : \begin{cases} x^2 + y^2 + 2y \geq 0; \\ x^2 + y^2 + 4y \leq 0; \\ 0 \leq x \leq -\frac{y}{\sqrt{3}} \end{cases}$$

3.

$$\begin{aligned} x &= \sqrt{y}; x = 2\sqrt{y}; \\ x + \frac{1}{3}z &= 2; z = 0 \end{aligned}$$

4.

$$\begin{aligned} z &= 9\sqrt{x^2 + y^2}; \\ z &= 22 - x^2 - y^2 \end{aligned}$$

5.

$$16 \leq x^2 + y^2 + z^2 \leq 100; y \leq -\frac{x}{\sqrt{3}};$$

$$z \leq \sqrt{\frac{x^2 + y^2}{3}}; y \geq -x\sqrt{3}$$

6.

$$S : 5x^2 = 3y^2 + 3z^2 \quad (x \geq 0);$$

$$D : \left\{ \frac{y^2}{4} + z^2 \leq 1, x = 0 \right\}$$

7.

$$1 \leq \frac{x^2}{4} + y^2 \leq 100; x \geq 0; y \geq \frac{1}{2}x;$$

$$\mu = \frac{x}{y^2}$$

8.

$$L : \rho = a(1 - \cos \varphi), 0 \leq \varphi \leq \pi \quad (a > 0)$$

9.

$$\vec{F} = (x^2 + y^2)\vec{i} + (x^2 - y^2)\vec{j};$$

$$L : y = \begin{cases} x, 0 \leq x \leq 1 \\ 2 - x, 1 < x \leq 2 \end{cases},$$

$$M(2, 0), N(0, 0).$$

10.

$$\int_C e^x [(1 - \cos y) dx - (y - \sin y) dy];$$

$$C : \{x = 0, y = 1, y = x\}$$

11.

$$\vec{F} = (3x - 3y + 2z)\vec{i}, \quad p : 3x - 2y - 2z + 6 = 0$$

ВАРИАНТ 21

1.

$$\iint_D (8xy + 9x^2y^2) dx dy$$

$$D : x = 1; y = \sqrt[3]{x}; y = -x^3$$

2.

$$\iint_D \sqrt{9 - x^2 - y^2} dx dy$$

$$D : \begin{cases} x^2 + y^2 \leq 3x; \\ x^2 + y^2 \leq 3y \end{cases}$$

3.

$$x^2 + y^2 = 18; y = \sqrt{3x};$$

$$z = \frac{25}{11}x; y = 0; z = 0$$

4.

$$z = \frac{15}{2}\sqrt{x^2 + y^2}; z = \frac{17}{2} - x^2 - y^2$$

5.

$$16 \leq x^2 + y^2 + z^2 \leq 81; y \leq 0;$$

$$z \geq \sqrt{\frac{x^2 + y^2}{99}}; y \leq -x\sqrt{3}$$

6.

$$S : 2x + 3y + z = 6;$$

$$D : \left\{ \frac{x^2}{4} + \frac{y^2}{9} \leq 1, z = 0 \right\}$$

7.

$$1 \leq \frac{x^2}{4} + y^2 \leq 4; y \leq \frac{\sqrt{3}}{2}x; y \leq 0;$$

$$\mu = x^2y$$

8.

L — часть параболы $y = 2\sqrt{x}$, $0 \leq x \leq 1$.

9.

$$\vec{F} = x\vec{j};$$

$$L : x^2 + y^2 = a^2, x \geq 0,$$

$$M(0, -a), N(0, a).$$

10.

$$\int_C (2y + x^2) dx + (y^2x + y - x) dy;$$

$$C : \{x = y^2 - 1, x = 0\}$$

11.

$$\vec{F} = x\vec{i} + \vec{j} + (y + z)\vec{k}, \quad p : x + y + z - 2 = 0$$

ВАРИАНТ 22

1.

$$\iint_D y^2 \cdot e^{-\frac{xy}{2}} dx dy$$

$$D : x = 0; y = 1; y = \frac{x}{2}$$

2.

$$\iint_D \frac{1}{\sqrt{9 - x^2 - y^2}} dx dy$$

$$D : \begin{cases} x^2 + y^2 \leq 3x; \\ -\frac{x}{\sqrt{3}} \leq y \leq \frac{x}{\sqrt{3}} \end{cases}$$

3.

$$\begin{aligned} y &= 2\sqrt{x}; x + y = 3; \\ z &= 3x; z = 0 \end{aligned}$$

4.

$$z = \sqrt{25 - x^2 - y^2}; z = \sqrt{\frac{x^2 + y^2}{99}}$$

5.

$$64 \leq x^2 + y^2 + z^2 \leq 144;$$

$$z \geq -\sqrt{\frac{x^2 + y^2}{63}}; 0 \leq y \leq \frac{x}{\sqrt{3}}$$

6.

$$S : x^2 + y^2 = z^2 \quad (z \geq 0);$$

$$D : \{x^2 + y^2 \leq 2x, z = 0\}$$

7.

$$16 \leq \frac{x^2}{3} + \frac{y^2}{9} \leq 100; y \geq x; x \geq 0;$$

$$\mu = \frac{x}{y}$$

8.

$$L : y = \frac{x^2}{4} - \frac{\ln x}{2}, \quad 1 \leq x \leq 2.$$

9.

$$\vec{F} = (x^2 - y^2)\vec{i} + xy\vec{j};$$

$$L : \text{отрезок } MN,$$

$$M(1, 1), N(3, 4).$$

10.

$$\int_C (y - x)^2 dx - (x + y)^2 dy;$$

$$C : \{y = \cos x, y = 0, -\pi/2 \leq x \leq \pi/2\}$$

11.

$$\vec{F} = (2x + 3y - 3z)\vec{j}, \quad p : 2x - 3y + 2z - 6 = 0$$

ВАРИАНТ 23

1.

$$\iint_D (24xy + 18x^2y^2) dx dy$$

$$D : x = 1; y = x^3; y = -\sqrt[3]{x}$$

2.

$$\iint_D \frac{1}{(9 + x^2 + y^2)^2} dx dy$$

$$D : \begin{cases} 3x \leq x^2 + y^2 \leq 9; \\ y \geq 0; x \geq 0 \end{cases}$$

3.

$$y = \sqrt{3x}; x + y = 6;$$

$$z = 4y; z = 0$$

4.

$$z = \sqrt{64 - x^2 - y^2}; z = 1;$$

$$x^2 + y^2 = 60 \text{ (внутри цилиндра)}$$

5.

$$64 \leq x^2 + y^2 + z^2 \leq 196;$$

$$z \leq \sqrt{\frac{x^2 + y^2}{3}}; \frac{x}{\sqrt{3}} \leq y \leq 0$$

6.

$$S : x^2 + y^2 + z^2 = 25 \quad (y \geq 0);$$

$$D : \{x^2 + z^2 \leq 16, y = 0\}$$

7.

$$1 \leq \frac{x^2}{3} + \frac{y^2}{4} \leq 4; 0 \leq y \leq 2x;$$

$$\mu = x$$

8.

$$L : y = x^3, \quad 0 \leq x \leq 2.$$

9.

$$\vec{F} = y\vec{i} - (y + x^2)\vec{j};$$

$$L : y = 2x - x^2,$$

$$M(0, 0), N(2, 0).$$

10.

$$\int_C (x^2 + y^2) dx + (y^2 - x^2) dy;$$

$$C : \{x = -\sqrt{R^2 - y^2}, x = 0\}$$

11.

$$\vec{F} = x\vec{i} + (y - z)\vec{j} + 2\vec{k}, \quad p : x - 2y - 2z + 4 = 0$$

ВАРИАНТ 24

1.

$$\iint_D y^2 \cdot \cos xy \, dx \, dy$$

$$D : x = 0; y = \sqrt{\pi}; y = 2x$$

2.

$$\iint_D \frac{108}{(9 - x^2 - y^2)^2} \, dx \, dy$$

$$D : \begin{cases} 3x \leq x^2 + y^2 \leq \frac{9}{4}; \\ x \leq y \end{cases}$$

3.

$$y = \sqrt{3x}; x^2 + y^2 = 18;$$

$$y = 0; z = \frac{5}{11}x; z = 0$$

4.

$$z = \frac{21}{2} \sqrt{x^2 + y^2};$$

$$z = \frac{23}{2} - x^2 - y^2$$

5.

$$64 \leq x^2 + y^2 + z^2 \leq 196;$$

$$-\sqrt{\frac{x^2 + y^2}{3}} \leq z \leq \sqrt{\frac{x^2 + y^2}{15}}$$

6.

$$S : z = 1 - x^2 - y^2;$$

$$D : \{x^2 + y^2 \leq 1, z = 0\}$$

7.

$$1 \leq x^2 + \frac{y^2}{6} \leq 4; 0 \leq y \leq 3\sqrt{2}x;$$

$$\mu = y$$

8.

$$L : \begin{cases} x = 5(t - \sin t) \\ y = 5(1 - \cos t) \end{cases}, \quad 0 \leq t \leq \pi$$

9.

$$\vec{F} = \left(\frac{1}{\sqrt{x}}\right) \vec{i} - \left(\frac{1}{\sqrt{y}}\right) \vec{j};$$

$$L : \begin{cases} x = r \cos t \\ y = r \sin t \end{cases}, \quad x \geq 0, y \geq 0,$$

$$M(r, 0), N(0, r).$$

10.

$$\int_C (y - x) \, dx - (x + y) \, dy;$$

$$C : \left\{ y = b\sqrt{1 - \frac{x^2}{a^2}}, y = 0 \right\}, \quad a > 0, b > 0$$

11.

$$\vec{F} = (2x + 3y + 5z) \vec{j} + \vec{k}, \quad p : x + 3y + z - 3 = 0$$

ВАРИАНТ 25

1.

$$\iint_D (12xy + 27x^2y^2) dx dy$$

$$D : x = 1; y = x^2; y = -\sqrt[3]{x}$$

2.

$$\iint_D \frac{y^2}{x^2} dx dy$$

$$D : \begin{cases} x^2 + y^2 + 2x \leq 0; \\ x^2 + y^2 \geq 2 \end{cases}$$

3.

$$x + y = 3; y^2 = 4x;$$

$$z = -y; z = 0 \quad (z \geq 0)$$

4.

$$z = 3\sqrt{x^2 + y^2};$$

$$z = 10 - x^2 - y^2$$

5.

$$16 \leq x^2 + y^2 + z^2 \leq 100;$$

$$-\sqrt{\frac{x^2 + y^2}{3}} \leq z \leq \sqrt{\frac{x^2 + y^2}{35}}$$

6.

$$S : x = z^2 - y^2;$$

$$D : \{y^2 + z^2 \leq 4, x = 0\}$$

7.

$$1 \leq x^2 + \frac{y^2}{2} \leq 4; y \leq \frac{\sqrt{6}}{3}x; y \leq 0;$$

$$\mu = xy^2$$

8.

$$L : \begin{cases} x = 10 \cos^3 t \\ y = 10 \sin^3 t \end{cases}, \quad 0 \leq t \leq \frac{\pi}{2}$$

9.

$$\vec{F} = 2xy\vec{i} - y^2\vec{j};$$

$$L : \text{отрезок } MN,$$

$$M(0, 0), N(2, 1).$$

10.

$$\int_C xy^2 dx - x^2y dy;$$

$$C : \{x = \sqrt{a^2 - y^2}, x = 0\}$$

11.

$$\vec{F} = -\vec{i} + (x - z + y)\vec{k}, \quad p : x + 2y + 2z - 4 = 0$$

ВАРИАНТ 26

1.

$$\iint_D y^2 \cdot \sin \frac{xy}{2} dx dy$$

$$D : x = 0; y = \sqrt{\pi}; y = x$$

2.

$$\iint_D \frac{x}{y^2} dx dy$$

$$D : \begin{cases} 16 \leq x^2 + y^2 \leq 4\sqrt{2} \cdot y; \\ x \geq 0 \end{cases}$$

3.

$$\begin{aligned} x &= 3\sqrt{3y}; x = 2\sqrt{3y}; \\ z &= 0; z + y = 3 \end{aligned}$$

4.

$$z = \sqrt{9 - x^2 - y^2}; z = \sqrt{\frac{x^2 + y^2}{80}}$$

5.

$$\begin{aligned} 9 \leq x^2 + y^2 + z^2 \leq 81; y \leq 0; \\ 0 \leq z \leq \sqrt{\frac{x^2 + y^2}{24}}; y \leq \frac{x}{\sqrt{3}} \end{aligned}$$

6.

$$\begin{aligned} S : 2x + y + 3z = 6; \\ D : \left\{ \frac{x^2}{4} + \frac{z^2}{9} \leq 1, y = 0 \right\} \end{aligned}$$

7.

$$\begin{aligned} 2 \leq \frac{x^2}{5} + \frac{y^2}{3} \leq 5; 0 \leq y \leq \frac{3\sqrt{5}}{5}x; \\ \mu = \frac{32}{63}xy \end{aligned}$$

8.

$$L : \begin{cases} x = \ln t \\ y = \frac{1}{2}(t + \frac{1}{t}) \end{cases}, 1 \leq t \leq 2$$

9.

$$\vec{F} = 4xy\vec{i} - x^2\vec{j};$$

$$L : y = \frac{1}{4}x^2,$$

$$M(0, 0), N(2, 1).$$

10.

$$\int_C y^2 dx - x dy;$$

$$C : \{y = \cos x, y = 0, -\pi/2 \leq x \leq \pi/2\}$$

11.

$$\vec{F} = x\vec{i} + (z - x)\vec{j} + y\vec{k}, \quad p : 3x + 2y + 3z - 6 = 0$$

ВАРИАНТ 27

1.

$$\iint_D (8xy + 18x^2y^2) dx dy$$

$$D : x = 1; y = \sqrt[3]{x}; y = -x^2$$

2.

$$\iint_D (9 - x^2 - y^2) dx dy$$

$$D : \begin{cases} x^2 + y^2 \leq 2y; \\ x \leq y \leq x\sqrt{3} \end{cases}$$

3.

$$y = \sqrt{5x}; x^2 + y^2 = 50$$

$$z = \frac{12}{25}x; z = 0 \quad (z \geq 0)$$

4.

$$z = \sqrt{100 - x^2 - y^2}; z = 6;$$

$$x^2 + y^2 = 51 \text{ (внутри цилиндра)}$$

5.

$$49 \leq x^2 + y^2 + z^2 \leq 169; y \geq 0;$$

$$-\sqrt{\frac{x^2 + y^2}{24}} \leq z \leq 0; y \geq \frac{x}{\sqrt{3}}$$

6.

$$S : x^2 + y^2 + z^2 = 1 \quad (z \geq 0);$$

$$D : \{x^2 + y^2 \leq x, z = 0\}$$

7.

$$1 \leq \frac{x^2}{3} + 3y^2 \leq 15; x \geq 0; y \geq \frac{1}{3}x;$$

$$\mu = \frac{x}{y}$$

8.

$$L : \begin{cases} x = e^t(\cos t + \sin t) \\ y = e^t(\cos t - \sin t) \end{cases}, 0 \leq t \leq \frac{\pi}{4}$$

9.

$$\vec{F} = 2xy\vec{i} - x^2\vec{j};$$

$$L : x = 2y^2,$$

$$M(0, 0), N(2, 1).$$

10.

$$\int_C e^x [(1 + \cos y) dx - (y + \sin y) dy];$$

$$C : \{y = x, x = 2, y = 1\}$$

11.

$$\vec{F} = \vec{i} + (5x + 2y + 3z)\vec{k}, \quad p : x + y + 3z - 3 = 0$$

ВАРИАНТ 28

1.

$$\iint_D y^2 \cdot e^{-\frac{xy}{8}} dx dy$$

$$D : x = 0; y = 4; y = 2x$$

2.

$$\iint_D (x - y) dx dy$$

$$D : \begin{cases} 2x \leq x^2 + y^2 \leq 4x; \\ x \geq 0; y \geq 0 \end{cases}$$

3.

$$x = 19\sqrt{2y}; x = 4\sqrt{2y};$$

$$z = 0; y + z = 2$$

4.

$$z = \sqrt{36 - x^2 - y^2};$$

$$z = \sqrt{\frac{x^2 + y^2}{63}}$$

5.

$$9 \leq x^2 + y^2 + z^2 \leq 64; y \leq \frac{x}{\sqrt{3}};$$

$$z \geq \sqrt{\frac{x^2 + y^2}{99}}; y \leq -\frac{x}{\sqrt{3}}$$

6.

$$S : x^2 + y^2 + z^2 = 4 \quad (x \geq 0);$$

$$D : \{z^2 + y^2 \leq 2z, x = 0\}$$

7.

$$1 \leq \frac{x^2}{3} + \frac{y^2}{1} \leq 9; -x \leq y \leq 0;$$

$$\mu = \frac{20}{363}xy^2$$

8.

$$L : \begin{cases} x = \cos t + t \sin t \\ y = \sin t - t \cos t \end{cases}, 0 \leq t \leq \frac{\pi}{2}$$

9.

$$\vec{F} = 2yx\vec{i} - y^2\vec{j};$$

L — ломаная линия, первое звено которой соединяет точки $M(0, 0)$ и $B(2, 0)$, а второе — точки $B(2, 0)$ и $N(2, 1)$.

10.

$$\int_C \frac{y}{x} dx + 2 \ln x dy;$$

$$C : \{2x + y = 4, x = 1, y = 0\}$$

11.

$$\vec{F} = x\vec{i} + \vec{j} + (y + z)\vec{k}, \quad p : 2x + y + 2z - 4 = 0$$

ВАРИАНТ 29

1.

$$\iint_D (12xy + 27x^2y^2) dx dy$$

$$D : x = 1; y = x^3; y = -\sqrt{x}$$

2.

$$\iint_D \ln \sqrt{9 - x^2 - y^2} dx dy$$

$$D : \begin{cases} 1 \leq x^2 + y^2 \leq 4; \\ y \geq x; y \geq \frac{x}{\sqrt{3}} \end{cases}$$

3.

$$x = \frac{5}{3}\sqrt{y}; x = \frac{5}{9}y;$$

$$z = 0; z = \frac{5}{9}(3 + \sqrt{y})$$

4.

$$z = \sqrt{81 - x^2 - y^2}; z = 5;$$

$$x^2 + y^2 = 45 \text{ (внутри цилиндра)}$$

5.

$$36 \leq x^2 + y^2 + z^2 \leq 100;$$

$$z \geq -\sqrt{\frac{x^2 + y^2}{63}}; -\frac{x}{\sqrt{3}} \leq y \leq x\sqrt{3}$$

6.

$$S : x^2 + y^2 + z^2 = 4 \quad (y \geq 0);$$

$$D : \{x^2 + z^2 \leq 2x, y = 0\}$$

7.

$$9 \leq x^2 + \frac{y^2}{3} \leq 100; 0 \leq y \leq 3x;$$

$$\mu = \frac{y}{x^2}$$

8.

$$L : \begin{cases} x = t^3 + 1 \\ y = t^2 \end{cases}, \quad 0 \leq t \leq 1$$

9.

$$\vec{F} = \cos y \cdot \vec{i} - \sin x \cdot \vec{j};$$

$$L : \text{отрезок } MN,$$

$$M(2, -2), N(-2, 2).$$

10.

$$\int_C (x^2y + x - y) dx + (y^2 + 2x) dy;$$

$$C : \{y = x^2 + 1, y = 2\}$$

11.

$$\vec{F} = (z + x)\vec{i} + y\vec{j} + \vec{k}, \quad p : 2x + 2y + z - 4 = 0$$

ВАРИАНТ 30

1.

$$\iint_D y^2 \cdot \cos \frac{xy}{2} dx dy$$

$$D : x = 0; y = \sqrt{2\pi}; y = 2x$$

2.

$$\iint_D e^{x^2+y^2} dx dy$$

$$D : \begin{cases} 0 \leq y \leq x; \\ 4 \leq x^2 + y^2 \leq 9 \end{cases}$$

3.

$$x = \sqrt{2y}; x + y = 4;$$

$$z = 0; z = \frac{3}{5}x$$

4.

$$z = \sqrt{100 - x^2 - y^2}; z = 7;$$

$$x^2 + y^2 = 51 \text{ (внутри цилиндра)}$$

5.

$$36 \leq x^2 + y^2 + z^2 \leq 144;$$

$$z \leq \sqrt{\frac{x^2 + y^2}{3}}; x\sqrt{3} \leq y \leq \frac{x}{\sqrt{3}}$$

6.

$$S : y = x^2 - z^2;$$

$$D : \{x^2 + z^2 \leq 9, y = 0\}$$

7.

$$1 \leq \frac{x^2}{3} + \frac{y^2}{4} \leq 4; x \geq 0; y \geq 2x;$$

$$\mu = \frac{40}{31}x^2y$$

8.

$$L : \begin{cases} x = 2(2 \cos t - \cos 2t) \\ y = 2(2 \sin t - \sin 2t) \end{cases}, 0 \leq t \leq \frac{\pi}{3}$$

9.

$$\vec{F} = -\frac{y^2}{x^{\frac{5}{3}} + y^{\frac{5}{3}}}\vec{i} + \frac{x^2}{x^{\frac{5}{3}} + y^{\frac{5}{3}}}\vec{j};$$

$$L : \begin{cases} x = R \cos^3 t \\ y = R \sin^3 t \end{cases}, x \geq 0, y \geq 0,$$

$$M(R, 0), N(0, R).$$

10.

$$\int_C (x + y)^2 dx - (x - y)^2 dy;$$

$$C : \{y = \sin x, y = 0, 0 \leq x \leq \pi\}$$

11.

$$\vec{F} = 2\vec{i} + (y + z)\vec{j} + x\vec{k}, \quad p : 2x + 2y - z - 2 = 0$$