

ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ

Задача 1. Найти общий интеграл дифференциального уравнения.

1.1. $4x dx - 3y dy = 3x^2 y dy - 2xy^2 dx.$

1.2. $x\sqrt{1+y^2} + yy'\sqrt{1+x^2} = 0.$

1.3. $\sqrt{4+y^2} dx - y dy = x^2 y dy.$

1.4. $\sqrt{3+y^2} dx - y dy = x^2 y dy.$

1.5. $6x dx - 6y dy = 2x^2 y dy - 3xy^2 dx.$

1.6. $x\sqrt{3+y^2} dx + y\sqrt{2+x^2} dy = 0.$

1.7. $(e^{2x} + 5) dy + y e^{2x} dx = 0.$

1.8. $y' y \sqrt{\frac{1-x^2}{1-y^2}} + 1 = 0.$

1.9. $6x dx - 6y dy = 3x^2 y dy - 2xy^2 dx.$

1.10. $x\sqrt{5+y^2} dx + y\sqrt{4+x^2} dy = 0.$

1.11. $y(4+e^x) dy - e^x dx = 0.$

1.12. $\sqrt{4-x^2} y' + xy^2 + x = 0.$

1.13. $2x dx - 2y dy = x^2 y dy - 2xy^2 dx.$

1.14. $x\sqrt{4+y^2} dx + y\sqrt{1+x^2} dy = 0.$

1.15. $(e^x + 8) dy - y e^x dx = 0.$

1.16. $\sqrt{5+y^2} + y' y \sqrt{1-x^2} = 0.$

1.17. $6x dx - y dy = yx^2 dy - 3xy^2 dx.$

1.18. $y \ln y + xy' = 0.$

1.19. $(1+e^x) y' = y e^x.$

1.20. $\sqrt{1-x^2} y' + xy^2 + x = 0.$

1.21. $6x dx - 2y dy = 2yx^2 dy - 3xy^2 dx.$

1.22. $y(1+\ln y) + xy' = 0.$

1.23. $(3+e^x) yy' = e^x.$

1.24. $\sqrt{3+y^2} + \sqrt{1-x^2} yy' = 0.$

1.25. $x dx - y dy = yx^2 dy - xy^2 dx.$

1.26. $\sqrt{5+y^2} dx + 4(x^2 y + y) dy = 0.$

1.27. $(1+e^x) yy' = e^x.$

1.28. $3(x^2 y + y) dy + \sqrt{2+y^2} dx = 0.$

1.29. $2x dx - y dy = yx^2 dy - xy^2 dx.$

1.30. $2x + 2xy^2 + \sqrt{2-x^2} y' = 0.$

1.31. $20x dx - 3y dy = 3x^2 y dy - 5xy^2 dx.$

Задача 2. Найти общий интеграл дифференциального уравнения.

2.1. $y' = \frac{y^2}{x^2} + 4\frac{y}{x} + 2.$

2.2. $xy' = \frac{3y^3 + 2yx^2}{2y^2 + x^2}.$

$$2.3. y' = \frac{x+y}{x-y}$$

$$2.5. 2y' = \frac{y^2}{x^2} + 6\frac{y}{x} + 3$$

$$2.7. y' = \frac{x+2y}{2x-y}$$

$$2.9. 3y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 4$$

$$2.11. y' = \frac{x^2 + xy - y^2}{x^2 - 2xy}$$

$$2.13. y' = \frac{y^2}{x^2} + 6\frac{y}{x} + 6$$

$$2.15. y' = \frac{x^2 + 2xy - y^2}{2x^2 - 2xy}$$

$$2.17. 2y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 8$$

$$2.19. y' = \frac{x^2 + 3xy - y^2}{3x^2 - 2xy}$$

$$2.21. y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 12$$

$$2.23. y' = \frac{x^2 + xy - 3y^2}{x^2 - 4xy}$$

$$2.25. 4y' = \frac{y^2}{x^2} + 10\frac{y}{x} + 5$$

$$2.27. y' = \frac{x^2 + xy - 5y^2}{x^2 - 6xy}$$

$$2.29. 3y' = \frac{y^2}{x^2} + 10\frac{y}{x} + 10$$

$$2.4. xy' = \sqrt{x^2 + y^2} + y$$

$$2.6. xy' = \frac{3y^3 + 4yx^2}{2y^2 + 2x^2}$$

$$2.8. xy' = 2\sqrt{x^2 + y^2} + y$$

$$2.10. xy' = \frac{3y^3 + 6yx^2}{2y^2 + 3x^2}$$

$$2.12. xy' = \sqrt{2x^2 + y^2} + y$$

$$2.14. xy' = \frac{3y^3 + 8yx^2}{2y^2 + 4x^2}$$

$$2.16. xy' = 3\sqrt{x^2 + y^2} + y$$

$$2.18. xy' = \frac{3y^3 + 10yx^2}{2y^2 + 5x^2}$$

$$2.20. xy' = 3\sqrt{2x^2 + y^2} + y$$

$$2.22. xy' = \frac{3y^3 + 12yx^2}{2y^2 + 6x^2}$$

$$2.24. xy' = 2\sqrt{3x^2 + y^2} + y$$

$$2.26. xy' = \frac{3y^3 + 14yx^2}{2y^2 + 7x^2}$$

$$2.28. xy' = 4\sqrt{x^2 + y^2} + y$$

$$2.30. xy' = 4\sqrt{2x^2 + y^2} + y$$

$$31. y' = \frac{x^2 + 2xy - 5y^2}{2x^2 - 6xy}$$

Задача 3. Найти решение задачи Коши.

$$3.1. y' - y/x = x^2, \quad y(1) = 0.$$

$$3.2. y' - y \operatorname{ctg} x = 2x \sin x, \quad y(\pi/2) = 0.$$

$$3.3. y' + y \cos x = \frac{1}{2} \sin 2x, \quad y(0) = 0.$$

$$3.3. y' + y \operatorname{tg} x = \cos^2 x, \quad y(\pi/4) = 1/2.$$

$$3.5. y' - \frac{y}{x+2} = x^2 + 2x, \quad y(-1) = 3/2.$$

$$3.6. y' - \frac{1}{x+1} y = e^x(x+1), \quad y(0) = 1.$$

$$3.7. y' - \frac{y}{x} = x \sin x, \quad y\left(\frac{\pi}{2}\right) = 1.$$

$$3.8. y' + \frac{y}{x} = \sin x, \quad y(\pi) = \frac{1}{\pi}.$$

$$3.9. y' + \frac{y}{2x} = x^2, \quad y(1) = 1.$$

$$3.10. y' + \frac{2x}{1+x^2} y = \frac{2x^2}{1+x^2}, \quad y(0) = \frac{2}{3}.$$

$$3.11. y' - \frac{2x-5}{x^2} y = 5, \quad y(2) = 4.$$

$$3.12. y' + \frac{y}{x} = \frac{x+1}{x} e^x, \quad y(1) = e.$$

$$3.13. y' - \frac{y}{x} = -2 \frac{\ln x}{x}, \quad y(1) = 1.$$

$$3.13. y' - \frac{y}{x} = -\frac{12}{x^3}, \quad y(1) = 4.$$

$$3.15. y' + \frac{2}{x} y = x^3, \quad y(1) = -5/6.$$

$$3.16. y' + \frac{y}{x} = 3x, \quad y(1) = 1.$$

$$3.17. y' - \frac{2xy}{1+x^2} = 1+x^2, \quad y(1) = 3.$$

$$3.18. y' + \frac{1-2x}{x^2} y = 1, \quad y(1) = 1.$$

$$3.19. y' + \frac{3y}{x} = \frac{2}{x^3}, \quad y(1) = 1.$$

$$3.20. y' + 2xy = -2x^3, \quad y(1) = e^{-1}.$$

$$3.21. y' + \frac{xy}{2(1-x^2)} = \frac{x}{2}, \quad y(0) = \frac{2}{3}.$$

$$3.22. y' + xy = -x^3, \quad y(0) = 3.$$

$$3.23. y' - \frac{2}{x+1} y = e^x(x+1)^2, \quad y(0) = 1. \quad 3.23. y' + 2xy = x e^{-x^2} \sin x, \quad y(0) = 1.$$

$$3.25. y' - 2y/(x+1) = (x+1)^3, \quad y(0) = 1/2. \quad 3.26. y' - y \cos x = -\sin 2x, \quad y(0) = 3.$$

$$27. y' - 4xy = -4x^3, \quad y(0) = -1/2.$$

$$3.28. y' - \frac{y}{x} = -\frac{\ln x}{x}, \quad y(1) = 1.$$

$$3.29. y' - 3x^2y = x^2(1+x^3)/3, \quad y(0) = 0.$$

$$3.30. y' - y \cos x = \sin 2x, \quad y(0) = -1.$$

$$3.31. y' - y/x = -2/x^2, \quad y(1) = 1.$$

Задача 4. Найти решение задачи Коши.

$$4.1. y' + xy = (1+x)e^{-x}y^2, \quad y(0) = 1.$$

$$4.2. xy' + y = 2y^2 \ln x, \quad y(1) = 1/2.$$

$$4.3. 2(xy' + y) = xy^2, \quad y(1) = 2.$$

$$4.4. y' + 4x^3y = 4(x^3+1)e^{-4x}y^2, \quad y(0) = 1.$$

$$4.5. xy' - y = -y^2(\ln x + 2) \ln x, \quad y(1) = 1.$$

$$4.6. 2(y' + xy) = (1+x)e^{-x}y^2, \quad y(0) = 2.$$

$$4.7. 3(xy' + y) = y^2 \ln x, \quad y(1) = 3.$$

$$4.8. 2y' + y \cos x = y^{-1} \cos x (1 + \sin x), \quad y(0) = 1.$$

$$4.9. y' + 4x^2y = 4y^2 e^{4x} (1 - x^3), \quad y(0) = -1.$$

$$4.10. 3y' + 2xy = 2xy^{-2} e^{-2x^2}, \quad y(0) = -1.$$

$$4.11. 2xy' - 3y = -(5x^2 + 3)y^3, \quad y(1) = 1/\sqrt{2}.$$

$$4.12. 3xy' + 5y = (4x - 5)y^4, \quad y(1) = 1.$$

$$4.13. 2y' + 3y \cos x = e^{2x} (2 + 3 \cos x) y^{-1}, \quad y(0) = 1.$$

$$4.14. 3(xy' + y) = xy^2, \quad y(1) = 3.$$

$$4.15. y' - y = 2xy^2, \quad y(0) = 1/2.$$

$$4.14. 2xy' - 3y = -(20x^2 + 12)y^3, \quad y(1) = 1/2\sqrt{2}.$$

$$4.17. y' + 2xy = 2x^3y^3, \quad y(0) = \sqrt{2}.$$

$$4.18. xy' + y = y^2 \ln x, \quad y(1) = 1.$$

$$4.19. 2y' + 3y \cos x = (8 + 12 \cos x) e^{2x} y^{-1}, \quad y(0) = 2.$$