

20.	$z = e^{y/x}$	$\frac{\partial}{\partial x} \left( x \cdot \frac{\partial z}{\partial x} \right) - y^2 \cdot \frac{\partial^2 z}{\partial y^2} = 0$
21.	$z = e^{-\cos(ax+y)}$	$a^2 \cdot \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial x^2}$
22.	$z = \ln(x^2 + y^2 + 2y + 1)$	$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$
23.	$z = \sin^2(y - ax)$	$a^2 \cdot \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial x^2}$
24.	$z = \frac{y}{x}$	$x^2 \cdot \frac{\partial^2 z}{\partial x^2} + 2xy \cdot \frac{\partial^2 z}{\partial x \partial y} + y^2 \cdot \frac{\partial^2 z}{\partial y^2} = 0$
25.	$z = y \cdot \sqrt{\frac{y}{x}}$	$x^2 \cdot \frac{\partial^2 z}{\partial x^2} - y^2 \cdot \frac{\partial^2 z}{\partial y^2} = 0$
26.	$z = \sqrt{\frac{x}{y}}$	$x^2 \cdot \frac{\partial^2 z}{\partial x^2} - \frac{2}{y} \left( y^2 \cdot \frac{\partial^2 z}{\partial y} \right) = 0$
27.	$z = \operatorname{arctg} \frac{x}{y}$	$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$
28.	$z = e^{xy}$	$x \cdot \frac{\partial z}{\partial x} + y \cdot \frac{\partial z}{\partial y} - 2 \cdot \frac{\partial^2 z}{\partial x \partial y} = -2z$
29.	$z = x^y$	$y \cdot \frac{\partial^2 z}{\partial x \partial y} = (1 + y \ln x) \cdot \frac{\partial z}{\partial x}$

ЗАДАЧА 2. Дана функция  $Z = f(x, y)$  и точки  $A(x_0, y_0)$ ,  $B(x_1, y_1)$ .  
 Требуется вычислить: 1) точное значение данной функции в точке B;  
 2) приближенное значение данной функции в точке B, используя формулу (6); 3) оценить в процентах относительную погрешность.

N кар.	$Z = f(x, y)$	$A(x_0, y_0)$	$B(x_1, y_1)$
1.	$z = 2x^2 + 3xy + 4y^2$	$A(1; 2)$	$B(1,03; 1,94)$
2.	$z = 2xy + 3x - 2y$	$A(2; 2)$	$B(1,93; 2,05)$
3.	$z = 2y^2 - 3xy + 4x$	$A(1; 3)$	$B(1,07; 2,94)$
4.	$z = x^2 - y^2 - 2x + y$	$A(4; 1)$	$B(3,98; 1,06)$
5.	$z = x^2 + y^2 + 2x + 3y$	$A(1; 2)$	$B(1,05; 1,98)$
6.	$z = y^2 + 6xy - 3y$	$A(3; 2)$	$B(2,94; 2,05)$

7.	$Z = 3xy + 2x + y$	$A(1; 2)$	$B(1,05; 1,93)$
8.	$Z = x^2 + y^2 + 5x - 3y$	$A(3; 2)$	$B(3,04; 1,95)$
9.	$Z = x^2 + xy + y^2$	$A(1; 2)$	$B(1,02; 1,96)$
10.	$Z = 3x^2 - xy + x + y$	$A(1, 3)$	$B(1,06; 2,92)$
11.	$Z = y^2 - xy - x^2$	$A(-4; 5)$	$B(-3,92; 5,06)$
12.	$Z = x^2 + 3xy - 6x$	$A(4; 1)$	$B(3,96; 1,03)$
13.	$Z = x^2 - y^2 + 6x + 3y$	$A(2; 3)$	$B(2,02; 2,97)$
14.	$Z = x^2 + 2xy + 3y^2$	$A(2; 1)$	$B(1,96; 1,04)$
15.	$Z = x^2 + y^2 + 2x + y - 1$	$A(2; 4)$	$B(1,98; 3,91)$
16.	$Z = 3x^2 + 2y^2 - xy$	$A(-1; 3)$	$B(-0,98; 2,97)$
17.	$Z = x^2 - y^2 + 5x + 4y$	$A(3; 2)$	$B(3,02; 1,98)$
18.	$Z = 2xy + 3y^2 - 5x$	$A(3; 4)$	$B(3,04; 3,95)$
19.	$Z = xy + 2y^2 - 2x$	$A(1; 2)$	$B(0,97; 2,03)$
20.	$Z = x^2 + y^2 - x - y$	$A(1; -3)$	$B(1,08; -2,94)$
21.	$Z = xy + y^2 - 2x$	$A(2; 1)$	$B(2,03; 0,96)$
22.	$Z = x^2 + y^2 - x + y$	$A(-2; 2)$	$B(-2,02; 2,05)$
23.	$Z = 2x^2 + 2xy - y^2$	$A(1; 3)$	$B(0,95; 2,94)$
24.	$Z = x^2 + 3xy - y^2$	$A(1; 3)$	$B(0,96; 2,95)$
25.	$Z = xy + 2x - y$	$A(2; 2)$	$B(1,93; 2,05)$
26.	$Z = 3y^2 - 9xy + y$	$A(1; 3)$	$B(1,07; 2,94)$
27.	$Z = xy + x - y$	$A(1,5; 2,3)$	$B(1,43; 2,35)$
28.	$Z = x^2 + 2xy + 3y^2$	$A(2; 1)$	$B(1,95; 1,04)$
29.	$Z = 2xy + 3y^2 - 5x$	$A(3; 4)$	$B(3,04; 3,95)$

ЗАДАЧА 3. Найти наибольшее и наименьшее значения функции  $Z = f(x, y)$  в области  $D$ .

N вар	$Z = f(x, y)$	Область $D$	N вар	$Z = f(x, y)$	Область $D$
1.	$Z = 6xy - 9x^2 - 9y^2 + 4x + 4y$	$0 \leq x \leq 1,$ $0 \leq y \leq 2$	16.	$Z = 10 + 2xy - x^2$	$0 \leq y \leq 4 - x^2$
2.	$Z = x^2 + xy - 2$	$y = 4x^2 - 4,$ $y = 0$	17.	$Z = x^2 + 2xy - y^2 + 4x$	$x = 0,$ $y = 0,$ $x + y + 2 = 0$
3.	$2Z = 4x^2 + 4xy - y^2 - 8y$	$y = 2x,$ $y = 2,$ $x = 0$	18.	$Z = x^2 + xy - 2$	$4x^2 - 4 = y,$ $y \leq 0$
4.	$Z = x^2 + 2xy + 4x - y^2$	$x = 0,$ $y = 0,$ $x + y + 2 = 0$	19.	$Z = x^2 + xy$	$-1 \leq x \leq 1$ $0 \leq y \leq 3$
5.	$Z = 5x^2 - 3xy + y^2$	$-1 \leq x \leq 1,$ $-1 \leq y \leq 1$	20.	$Z = x^3 + 8y^3 - 6xy + 1$	$y = 1,$ $y = -1,$ $x = 0, x = 2$
6.	$Z = \frac{1}{2}(x^2 - 2xy)$	$y = 2x^2,$ $y = 8$	21.	$Z = x^2 - xy + y^2 - 4x$	$x = 0,$ $y = 0,$ $2x + 3y = 12$
7.	$Z = 3x + y - xy$	$y = x,$ $y = 4,$ $x = 0$	22.	$Z = x^2 + 3y^2 + x - y$	$x = 1,$ $y = 1,$ $x + y = 1$
8.	$Z = xy - 3x - 2y$	$0 \leq y \leq 4,$ $0 \leq x \leq 4$	23.	$Z = x^2 - 2y^2 + 4xy - 6x - 1$	$x = 0,$ $y = 0,$ $x + y = 3$
9.	$Z = x^2 + y^2 - 9xy + 27$	$0 \leq x \leq 3$ $0 \leq y \leq 3$	24.	$Z = xy - 2x - y$	$0 \leq x \leq 3$ $0 \leq y \leq 4$
10.	$Z = x^2 + 2y^2 + 1$	$x \geq 0,$ $y \geq 0,$ $x + y = 3$	25.	$Z = \frac{1}{2}x^2 - xy$	$y = \frac{1}{3}x^2,$ $y = 3$
11.	$Z = x^2 + y^2 - xy + x + y$	$x = 0,$ $y = 0,$ $x + y = -3$	26.	$Z = 2x + y - xy$	$0 \leq x \leq 4,$ $0 \leq y \leq 4$
12.	$Z = 3 - 2x^2 - xy - y^2$	$x = 1,$ $y \geq 0,$ $y = x$	27.	$Z = x^2 + 2xy + 4x + 8y$	$0 \leq x \leq 1$ $0 \leq y \leq 2$
13.	$Z = x^2 + 3y^2 + x - y$	$x \geq 1,$ $y = -1,$ $x + y = 1$	28.	$Z = x^2 + 2y - 3x - y$	$0 \leq x \leq 2,$ $0 \leq y \leq 3$
14.	$Z = x^2 + 2xy + 2y^2$	$-1 \leq x \leq 1,$ $0 \leq y \leq 2$	29.	$Z = xy - x - 2y$	$y = 2,$ $x = 3,$ $y = 0$
15.	$Z = 5x^2 - 3xy + y^2 + 4$	$x \geq -1,$ $y = -1,$ $x + y = 1$			

ЗАДАЧА 4. Найти экстремум функции  $Z = f(x, y)$  при условии  $\varphi(x, y) = 0$

$N$ вар	$Z = f(x, y)$	$\varphi(x, y) = 0$	$N$ вар	$Z = f(x, y)$	$\varphi(x, y) = 0$
1.	$Z = xy$	$x + y = 1$	16.	$Z = x^2 + y^2$	$x - y = 1$
2.	$Z = xy$	$\frac{x}{2} + \frac{y}{2} = 1$	17.	$Z = \frac{1}{x} + \frac{1}{y}$	$x + y = 2$
3.	$Z = xy$	$\frac{x}{4} + \frac{y}{2} = 1$	18.	$Z = \frac{x^2 + y^2 - xy + x + y - 4}{x + y - 4}$	$x + y + 3 = 0$
4.	$Z = xy$	$2x + y = 4$	19.	$Z = \frac{x - y - 4}{\sqrt{2}}$	$x^2 + y^2 = 1$
5.	$Z = xy$	$x + 3y = 6$	20.	$Z = xy^2$	$x + 2y = 1$
6.	$Z = \frac{x}{3} + \frac{y}{4}$	$x^2 + y^2 = 1$	21.	$Z = 2x + y$	$x^2 + y^2 = 1$
7.	$Z = x - y$	$x^2 + y^2 = 1$	22.	$Z = xy$	$x^2 + y^2 = 1$
8.	$Z = 4x - 3y$	$x^2 + y^2 = 1$	23.	$Z = xy$	$x^2 + y^2 = 2$
9.	$Z = \frac{x}{4} - \frac{y}{3}$	$x^2 + y^2 = 25$	24.	$Z = x^2 - xy + y^2 - 4x$	$2x + 3y - 12 = 0$
10.	$Z = -\frac{x}{5} + \frac{y}{12}$	$x^2 + y^2 = 1$	25.	$Z = 1 - x^2 - y^2$	$(x-1)^2 + (y-1)^2 = 1$
11.	$Z = 12x - 5y$	$x^2 + y^2 = \frac{1}{169}$	26.	$Z = x^2 + y^2$	$\frac{x^2}{4} + \frac{y^2}{9} = 1$
12.	$Z = x^2 + y^2$	$x + y = 1$	27.	$Z = x^2 + y^2$	$\frac{x^2}{9} + \frac{y^2}{16} = 1$
13.	$Z = x^2 + y^2$	$\frac{x}{3} + \frac{y}{4} = 1$	28.	$Z = x^2 + y^2$	$x^2 + 4y^2 = 1$
14.	$Z = x^2 + y^2$	$4x - 3y = 1$	29.	$Z = x^2 + y^2$	$9x^2 + y^2 = 1$
15.	$Z = x^2 + y^2$	$-\frac{x}{4} + \frac{y}{3} = 1$	30.	$Z = x^2 + y^2$	$\frac{x^2}{16} - \frac{y^2}{4} = 1$