

Задачи для типовых расчетов

Задача 9.1. Найдите неопределенные интегралы.

1. $\int \sqrt[6]{(x+4)^5} dx.$

2. $\int \left(\frac{1-x}{x}\right)^2 dx.$

3. $\int \frac{dx}{\sqrt{2-2x^2}}.$

4. $\int \frac{dx}{3x^2-9}.$

5. $\int \frac{dx}{7x^2+7}.$

6. $\int \frac{(\arctg x)^3}{1+x^2} dx.$

7. $\int \frac{x^3}{4-x^2} dx.$

8. $\int (3x-7)^{17} dx.$

9. $\int (x-1)x^{-\frac{2}{3}} dx.$

10. $\int \frac{dx}{\sqrt[5]{3-2x}}.$

11. $\int \frac{dx}{4x^2 - 4}$

13. $\int \sin(2 - 4x) dx$

15. $\int \frac{dx}{\sqrt{(5x-1)^2 + 1}}$

17. $\int 4^{2-3x} dx$

19. $\int \frac{dx}{\sin^2(2-3x)}$

21. $\int \frac{dx}{\sin^2(1-2x)}$

23. $\int \frac{dx}{\sqrt{4x^2 + 3}}$

25. $\int \frac{dx}{\cos^2(1-3x)}$

27. $\int \frac{dx}{\sqrt{3-2x}}$

29. $\int \cos(3-5x) dx$

12. $\int \frac{dx}{\sqrt{8+4x^2}}$

14. $\int \cos(1-2x) dx$

16. $\int 2^{1-4x} dx$

18. $\int \frac{dx}{5+3x}$

20. $\int 2^{1-x} dx$

22. $\int \frac{dx}{\cos^2 4x}$

24. $\int \frac{dx}{\cos^2(1-3x)}$

26. $\int \sqrt{3-5x} dx$

28. $\int 4^{1-3x} dx$

30. $\int \sqrt[3]{2x+7} dx$

Задача 9.2. Найдите неопределенные интегралы.

1. $\int x \cdot \sqrt[5]{5-x^2} dx$

2. $\int \frac{x^3}{x^8+5} dx$

3. $\int (\sin 2x + \cos 2x)^2 dx$

4. $\int \frac{dx}{\sin^2 x \cdot \cos^2 x}$

5. $\int \frac{\cos x dx}{\sqrt[3]{\sin x}}$

6. $\int \operatorname{ctg}^2 x dx$

7. $\int \operatorname{tg} \frac{x}{3} dx$

8. $\int \operatorname{tg}^2 x dx$

9. $\int \cos^2 2x dx$

10. $\int \operatorname{ctg} \frac{x}{2} dx$

11. $\int \frac{2-3\operatorname{ctg}^2 x}{\cos^2 x} dx$

12. $\int \sin 3x \cdot \cos x dx$

13. $\int \operatorname{tg} x dx$

14. $\int \frac{\sin x - 4}{\cos^2 x} dx$

15. $\int \sin 2x \cdot \cos 2x dx$

16. $\int \sin 2x dx$

17. $\int \frac{\sin 2x dx}{1 + \cos 2x}$

18. $\int \operatorname{ctg} x dx$

19. $\int \frac{\cos 3x dx}{5 + 3\sin 3x}$

20. $\int \frac{\cos x + 1}{\sin^2 x} dx$

21. $\int \sin \frac{x}{2} \cos \frac{x}{2} dx.$

23. $\int \frac{\sin 2x dx}{3 \sin^2 x + 4}.$

25. $\int \sin \frac{3x}{2} \cos \frac{3x}{2} dx.$

27. $\int e^{\operatorname{ctg} x} \frac{dx}{\sin^2 x}.$

29. $\int x \cdot 3^{4-x^2} dx.$

22. $\int \frac{\sin x}{(1-3 \cos x)^3} dx.$

24. $\int \frac{\sin x dx}{\sqrt{\cos^2 x + 2}}.$

26. $\int e^{4-x^2} x dx.$

28. $\int \frac{1+x}{\sqrt{1-x^2}} dx.$

30. $\int \frac{dx}{\sqrt{3-x^2-2x}}.$

Задача 9.3. Найдите неопределенные интегралы.

1. $\int \sin^2 \frac{x}{2} dx.$

3. $\int x^2 e^{-x^3} dx.$

5. $\int \frac{1+\operatorname{tg} x}{\cos^2 x} dx.$

7. $\int x \cdot e^{x^2} dx.$

9. $\int \frac{e^x dx}{e^{2x} + 4}.$

11. $\int e^x \cdot \sin(e^x) dx.$

13. $\int (e^x + e^{-x})^2 dx.$

15. $\int \frac{dx}{\sqrt{(1-x^2) \arcsin x}}.$

17. $\int \frac{x^3 dx}{\sqrt{9-x^8}}.$

19. $\int \frac{3^x dx}{\sqrt{9^x + 1}}.$

21. $\int \frac{x dx}{\sqrt{x^4 + 5}}.$

23. $\int \frac{\cos \frac{1}{x}}{x^2} dx.$

25. $\int \frac{3x dx}{\sqrt{x^4 + 7}}.$

2. $\int e^{\sin x} \cos x dx.$

4. $\int 3^{\sin x} \cos x dx.$

6. $\int \frac{x^3 + x}{x^4 + 1} dx.$

8. $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx.$

10. $\int \frac{e^{-\operatorname{tg} x}}{\cos^2 x} dx.$

12. $\int \frac{1-\sqrt{x}}{\sqrt{x}(1+x)} dx.$

14. $\int \frac{x^3}{\sqrt{4-x^8}} dx.$

16. $\int \frac{e^x}{\sqrt{e^{2x} - 2}} dx.$

18. $\int \frac{x dx}{\sqrt{1-x^4}}.$

20. $\int \frac{x^2 dx}{\sqrt{2+x^6}}.$

22. $\int \frac{x^5 + x^2}{\sqrt{5+x^6}} dx.$

24. $\int \frac{x^5 dx}{x^{12} - 1}.$

26. $\int \frac{dx}{\sqrt{4x-x^2}}.$

27. $\int \sqrt[3]{3-x^3} x^2 dx.$

29. $\int \frac{x + \frac{\ln x}{x}}{x^2 + \ln^2 x} dx.$

Задача 9.4. Найдите неопределенные интегралы.

1. $\int \frac{(\operatorname{arctg} x)^3}{1+x^2} dx.$

3. $\int \operatorname{tg} x \cdot (\ln \cos x) dx.$

5. $\int \frac{1 - \cos x}{(x - \sin x)^2} dx.$

7. $\int \frac{2 \operatorname{arctg} x + x}{1+x^2} dx.$

9. $\int \frac{dx}{\cos^2 x \sqrt{4 - \operatorname{tg}^2 x}}.$

11. $\int \frac{x + \frac{1}{x}}{\sqrt{x^2 + 1}} dx.$

13. $\int \frac{dx}{x \ln^2 x}.$

15. $\int \frac{2x^2 - x^5}{1+x^6} dx.$

17. $\int \frac{dx}{x \sqrt{3 + \ln^2 x}}.$

19. $\int \frac{dx}{x(\ln^2 x + 4)}.$

21. $\int \frac{dx}{\arccos^5 x \sqrt{1-x^2}}.$

23. $\int \frac{1-2x}{\sqrt{1-4x^2}} dx.$

25. $\int \frac{dx}{\arcsin^4 x \sqrt{1-x^2}}.$

27. $\int \frac{e^x dx}{\sqrt{2+2e^x + e^{2x}}}.$

29. $\int \frac{x dx}{\sqrt{9-x^4}} dx.$

28. $\int \frac{\sin 2x}{3 + \sin^2 x} dx.$

30. $\int \frac{dx}{x \sqrt{\ln^2 x + 4 \ln x + 5}}.$

2. $\int \frac{\arccos^3 x - 1}{\sqrt{1-x^2}} dx.$

4. $\int \frac{\operatorname{tg}(x+1)}{\cos^2(x+1)} dx.$

6. $\int \frac{\sin 2x}{1 + \cos^2 x} dx.$

8. $\int \frac{x - \cos x}{x^2 - 2 \sin x} dx.$

10. $\int \frac{2x - \sqrt{\arcsin x}}{\sqrt{1-x^2}} dx.$

12. $\int \frac{1}{\cos^2(1 + \ln x) x} dx.$

14. $\int \frac{dx}{x(\ln x + 5)}.$

16. $\int \frac{\ln^3 x + 2}{x \ln x} dx.$

18. $\int \frac{2^{\operatorname{arctg} 2x}}{1+4x^2} dx.$

20. $\int \frac{dx}{x \sqrt{1 - \ln^2 x}}.$

22. $\int \frac{\sin \frac{1}{x}}{x^2} dx.$

24. $\int \frac{2 + \ln x}{x} dx.$

26. $\int \frac{x^3 + 2x}{x^4 - 4} dx.$

28. $\int \frac{dx}{x \sqrt{4 - \ln^2 x}}.$

30. $\int \frac{2^x dx}{4^x + 2^{x+1} + 1}.$

Задача 9.5. Найдите неопределенные интегралы.

1. $\int \frac{1 + \ln x}{x} dx.$
2. $\int \frac{dx}{\sqrt{3-x^2-4x}}.$
3. $\int \frac{e^{2x} dx}{e^x - 1}.$
4. $\int \cos^3 x \cdot \sin 2x dx.$
5. $\int \frac{e^{2x}}{\sqrt{e^x + 1}} dx.$
6. $\int \frac{dx}{\sqrt{4x-3-x^2}}.$
7. $\int \frac{\sin^5 x}{\cos^4 x} dx.$
8. $\int \frac{\ln 2x dx}{\ln 4x x}.$
9. $\int \frac{1 + \frac{1}{2\sqrt{x}}}{(\sqrt{x} + x)^2} dx.$
10. $\int \frac{dx}{3e^x + 1}.$
11. $\int \frac{dx}{\sqrt{6-4x-2x^2}}.$
12. $\int \frac{dx}{\sqrt{x^2-6x+3}}.$
13. $\int \frac{x dx}{\sqrt{3+x^4}}.$
14. $\int \frac{x-4}{\sqrt{x+2}} dx.$
15. $\int \frac{e^x - 1}{3e^x + 1} dx.$
16. $\int \frac{x dx}{\sqrt{x-1}}.$
17. $\int \frac{e^{3x} dx}{e^{2x} - 1}.$
18. $\int \frac{dx}{e^x + 1}.$
19. $\int \frac{x dx}{\sqrt[3]{x-1}}.$
20. $\int \frac{x dx}{\sqrt{2x-1}}.$
21. $\int \frac{e^{2x} - 2e^x}{e^{2x} + 1} dx.$
22. $\int \frac{e^{2x}}{e^x + 1} dx.$
23. $\int \frac{e^{2x}}{e^x - 1} dx.$
24. $\int \frac{dx}{e^x - 1}.$
25. $\int \frac{e^{2x} - 3e^x}{e^{2x} + 1} dx.$
26. $\int \frac{1}{x+1} \frac{dx}{\sqrt{x}}.$
27. $\int \frac{dx}{x \ln x \cdot \ln \ln x}.$
28. $\int \frac{dx}{\cos^2 x (4 + \operatorname{tg}^2 x)}.$
29. $\int \frac{dx}{\sqrt{5-4x+x^2}}.$
30. $\int \frac{x + \operatorname{arctg} 2x}{1+4x^2} dx.$

Задача 9.6. Найдите неопределенные интегралы.

1. $\int (6-5x)e^{-3x} dx.$
2. $\int \operatorname{arctg} \sqrt{8x-1} dx.$
3. $\int (6x-3)\cos 2x dx.$
4. $\int \ln(x^2+9) dx.$

5. $\int x \ln^2 x dx.$
7. $\int x \cos^2 x dx.$
9. $\int \frac{\ln x}{x^3} dx.$
11. $\int x \arcsin x dx.$
13. $\int x \cdot 3^{\frac{x}{2}} dx.$
15. $\int \frac{x dx}{\sin^2 x}.$
17. $\int \ln(x + \sqrt{1+x^2}) dx.$
19. $\int \arccos 2x dx.$
21. $\int \arcsin x dx.$
23. $\int \frac{x \cos x}{\sin^3 x} dx.$
25. $\int \arccos x dx.$
27. $\int (x^3 - 4x + 5) \cos 3x dx.$
29. $\int \ln(x+2)x^2 dx.$
6. $\int \frac{x dx}{\sin^2 x}.$
8. $\int x^2 \ln x dx.$
10. $\int x \operatorname{arctg} x dx.$
12. $\int \frac{\ln(\ln x)}{x} dx.$
14. $\int x \operatorname{tg}^2 x dx.$
16. $\int \sqrt{x} \ln x dx.$
18. $\int \frac{\arcsin \sqrt{x}}{\sqrt{1-x}} dx.$
20. $\int \frac{\arcsin \frac{x}{2}}{\sqrt{2-x}} dx.$
22. $\int x \ln \frac{1-x}{1+x} dx.$
24. $\int \operatorname{arctg} \frac{1}{x} dx.$
26. $\int x^3 \ln(x+1) dx.$
28. $\int e^{2x} \cos x dx.$
30. $\int \arccos x \cdot x dx.$

Задача 9.7. Найдите неопределенные интегралы.

1. $\int x^2 \cos 3x dx.$
3. $\int (x^2 - 2x + 5)e^{-x} dx.$
5. $\int x^3 \sin \frac{x}{3} dx.$
7. $\int x^2 e^{\frac{x}{2}} dx.$
9. $\int (x^2 - 2) \cos x dx.$
11. $\int (x^2 + 3x - 7)e^{-2x} dx.$
13. $\int x \operatorname{arctg}^2 x dx.$
15. $\int \left(1 - \frac{1}{x^2}\right) \ln^2(x^2 + 1) dx.$
17. $\int x^2 \sin x dx.$
2. $\int \ln^2 2x dx.$
4. $\int (x+1)^2 \cos x dx.$
6. $\int \sqrt[3]{x} \ln^2 x dx.$
8. $\int \frac{x^2 + 2}{e^{3x}} dx.$
10. $\int \frac{\ln^2 x}{x^2} dx.$
12. $\int (\arcsin x)^2 dx.$
14. $\int x^2 e^x dx.$
16. $\int x^2 \cos 2x dx.$
18. $\int x^2 \sin 2x dx.$

19. $\int x \ln^2 x \, dx.$
 21. $\int (x^2 + 1) \cdot 3^x \, dx.$
 23. $\int x \ln^2 x \, dx.$
 25. $\int (x^2 + 15) \cdot 5^x \, dx.$
 27. $\int \arccos^2 x \cdot x \, dx.$
 29. $\int (x^3 - 3x^2) e^{-3x} \, dx.$
20. $\int (x^2 + 1) e^{-2x} \, dx.$
 22. $\int \arcsin^2 x \, dx.$
 24. $\int \frac{\ln^2 x}{x^2} \, dx.$
 26. $\int (x^3 + 3x + 2) \sin 2x \, dx.$
 28. $\int (4 - 3x^2 + 2x^3) \sin 2x \, dx.$
 30. $\int \frac{\ln x \, dx}{x^2}.$

Задача 9.8. Найдите неопределенные интегралы.

1. $\int \frac{2x^2 - 3x + 3}{x^3 - 2x^2 + x} \, dx.$
 3. $\int \frac{x^2 \, dx}{(x+2)(x-1)^2}.$
 5. $\int \frac{dx}{x^3 - 2x^2 + x}.$
 7. $\int \frac{2x-5}{(x-2)^2(x+1)} \, dx.$
 9. $\int \left(\frac{x+2}{x-1} \right)^2 \, dx.$
 11. $\int \frac{dx}{x^4 - x^2}.$
 13. $\int \frac{x^3 + 6x^2 + 11x + 7}{(x^2 + 3x + 2)(x+2)} \, dx.$
 15. $\int \frac{x^4 - 3x^3 + 9x - 8}{x^3 - 4x^2 + 4x} \, dx.$
 17. $\int \frac{x^3 + x^2 - x + 2}{x^2(x-1)} \, dx.$
 19. $\int \frac{2x^4 + 8x^3 + x^2 + x - 20}{x^3(x+5)} \, dx.$
 21. $\int \frac{x^3 - 2x^2 - 12x - 7}{x^3 - 3x - 2} \, dx.$
 23. $\int \frac{x^4 + 2x^3 + 9x^2 + 5x + 2}{x^2(x+1)} \, dx.$
 25. $\int \frac{x^3 - 3x^2 + 7x - 1}{2x^3 - x^2 + 3} \, dx.$
2. $\int \frac{x^3 + 1}{x(x-1)^3} \, dx.$
 4. $\int \frac{x^2 \, dx}{(x+2)^2(x+1)}.$
 6. $\int \frac{dx}{x(x+1)^2}.$
 8. $\int \frac{dx}{x^3 + x^2}.$
 10. $\int \frac{x^3 + 1}{x^3 - x^2} \, dx.$
 12. $\int \frac{x^2 - 3x + 2}{x(x^2 + 2x + 1)} \, dx.$
 14. $\int \frac{x^5 - x^4 + 3x - 2}{x^4 - x^3} \, dx.$
 16. $\int \frac{x^4 - 3x^2 + 3x - 1}{x^3 - 3x - 2} \, dx.$
 18. $\int \frac{x^3 - 4x + 1}{x^3 - 2x^2 + x} \, dx.$
 20. $\int \frac{x^4 + 2x^3 - 2x + x^2 + 1}{(x+1)^2(x-1)} \, dx.$
 22. $\int \frac{2x^3 - 2x^2 - 16x + 32}{(x-2)(x^2 - 4)} \, dx.$
 24. $\int \frac{x^3 + 3x^2 + 8x + 12}{(x^2 + 4x + 4)(x-1)} \, dx.$
 26. $\int \frac{x^3 + x^2 + 2}{(x+1)(x^2 + 1)} \, dx.$

27. $\int \frac{x^3 - 1}{x^3 + 1} dx.$

29. $\int \frac{x^4 + 1}{(x^2 + 1)(x^2 - 1)} dx.$

28. $\int \frac{(x+1)dx}{x^3 - 6x^2 + 9x}.$

30. $\int \left(\frac{x+1}{x-1}\right)^2 \frac{dx}{x}.$

Задача 9.9. Найдите неопределенные интегралы.

1. $\int \frac{dx}{x(x^2 + 1)}.$

3. $\int \frac{x dx}{x^3 - 1}.$

5. $\int \frac{7x - 15}{x^3 - 2x^2 + 5x} dx.$

7. $\int \frac{x - 4}{(x - 2)(x^2 + 1)} dx.$

9. $\int \frac{x^3 + x + 1}{x(x^2 + 1)} dx.$

11. $\int \frac{x^3 + 3}{(x + 1)(x^2 + 1)} dx.$

13. $\int \frac{2x - 1}{x^3 - 1} dx.$

15. $\int \frac{x + 2}{x^3 - 2x^2 + 2x} dx.$

17. $\int \frac{x - 2}{x^3 + 4x} dx.$

19. $\int \frac{dx}{x^3 + x^2 + 2x + 2}.$

21. $\int \frac{dx}{x^3 - 8}.$

23. $\int \frac{dx}{x^4 - 1}.$

25. $\int \frac{dx}{x^3 - 64}.$

27. $\int \frac{3x + 1}{(x + 1)^2(x^2 + 4)} dx.$

29. $\int \frac{dx}{x(x + 2)^2}.$

2. $\int \frac{x}{x^3 + 1} dx.$

4. $\int \frac{(x + 1)^3 dx}{x^3 - 1}.$

6. $\int \frac{x^3 + x - 1}{(x^2 + 2)(x - 1)} dx.$

8. $\int \frac{x^3 + x^2 - 5}{x^3 - 8} dx.$

10. $\int \frac{dx}{(x^2 + 1)(x^2 + 4)}.$

12. $\int \frac{x^2}{1 - x^4} dx.$

14. $\int \frac{3x dx}{x^3 + x^2 + 2x + 2} dx.$

16. $\int \frac{7x - 15}{x^3 - 2x^2 + 5x} dx.$

18. $\int \frac{2x^2 + x + 4}{x^3 + x^2 + 4x + 4} dx.$

20. $\int \frac{3x^2 + 11x + 8}{(x + 2)(x^2 + 2x + 2)} dx.$

22. $\int \frac{x^3 - 2x + 5}{x^4 - 1} dx.$

24. $\int \frac{dx}{x^3 + 8}.$

26. $\int \frac{x^3 - 3x^2 + x - 4}{(x^2 + 4)(x^2 + 9)} dx.$

28. $\int \frac{x^4 dx}{x^4 - 2x^2 + 1}.$

30. $\int \frac{dx}{x^2(x^2 - x + 1)}.$

Задача 9.10. Найдите неопределенные интегралы.

1. $\int \frac{\sqrt{x + 9}}{x} dx.$

2. $\int x \cdot \sqrt{x + 4} dx.$

3. $\int \frac{dx}{\sqrt{x+1}}$
5. $\int x \cdot \sqrt{x-2} dx$
7. $\int \frac{x+\sqrt{1+x}}{\sqrt[3]{1+x}} dx$
9. $\int \frac{dx}{\sqrt{2x-1}-\sqrt[3]{2x-1}}$
11. $\int \frac{\sqrt{x}}{x+2} dx$
13. $\int \frac{dx}{\sin^2 x + 4 \cos^2 x}$
15. $\int \frac{dx}{3+5 \cos x}$
17. $\int \frac{dx}{\sin^2 x + \operatorname{tg}^2 x}$
19. $\int \frac{\sin^2 x dx}{1 + \cos^2 x}$
21. $\int \frac{dx}{2 \sin x - 3 \cos x}$
23. $\int \frac{dx}{4 \sin x + 3 \cos x + 1}$
25. $\int \frac{dx}{\sin x - 3 \cos x}$
27. $\int \operatorname{tg}^5 x dx$
29. $\int \frac{\sqrt{x-1}+1}{\sqrt[3]{x-1}} dx$
4. $\int \frac{x dx}{\sqrt{x+3}}$
6. $\int \frac{dx}{\sqrt{x+\sqrt[3]{x}}}$
8. $\int \frac{x^3}{\sqrt{x-1}} dx$
10. $\int \frac{\sqrt{x}-1}{\sqrt[3]{x+1}} dx$
12. $\int \frac{\sqrt{x+4}}{x} dx$
14. $\int \frac{dx}{\sin^4 x \cos^2 x}$
16. $\int \frac{\cos 2x dx}{\sin^4 x}$
18. $\int \frac{dx}{1 + \sin^2 x}$
20. $\int \frac{1 + \cos x}{\sin^3 x} dx$
22. $\int \frac{dx}{\sin^3 x}$
24. $\int \frac{dx}{1 + 3 \sin^2 x}$
26. $\int \operatorname{tg}^4 x dx$
28. $\int \sin^4 2x dx$
30. $\int \frac{\sqrt{x+1}}{x^2} dx$

Задача 9.11. Вычислите определенные интегралы.

1. $\int_{\frac{\pi}{4}}^0 \frac{\sin^2 x dx}{\cos^2 x - 3 \sin^2 x}$
3. $\int_0^{\frac{\pi}{4}} \frac{1}{3 \cos^2 x - 5 \sin^2 x} dx$
5. $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos x + \sin x} dx$
2. $\int_0^{\frac{\pi}{2}} \frac{1}{1 - 2 \cos x + 3 \sin x} dx$
4. $\int_0^{\frac{\pi}{2}} \frac{\sin x}{(1 + \sin x)^2} dx$
6. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\cos x dx}{1 + \sin x - \cos x}$