

КОНТРОЛЬНАЯ РАБОТА № 4

I. Найти неопределенные интегралы. Правильность полученных результатов проверить дифференцированием.

1. 1) $\int \frac{xdx}{7+x^2};$

2) $\int \frac{(x+18)dx}{x^2-4x-12};$

3) $\int (3-x) \cos x dx.$

2. 1) $\int \frac{dx}{\sin^2 \frac{x}{5}};$

2) $\int \frac{(x+4)dx}{x^2-2x-8};$

3) $\int x \ln(1-3x) dx.$

3. 1) $\int \frac{dx}{\sqrt{5-x^2}};$

2) $\int \frac{(x+23)dx}{x^2+x-20};$

3) $\int xe^{-7x} dx.$

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| 4. 1) $\int \frac{dx}{3x+3};$ | 2) $\int \frac{(x+12)dx}{x^2-x-6};$ | 3) $\int \operatorname{arctg} 4x dx.$ |
| 5. 1) $\int \sin(2-3x) dx;$ | 2) $\int \frac{(x+19)dx}{x^2-2x-15};$ | 3) $\int \sqrt{x^3} \ln x dx.$ |
| 6. 1) $\int e^{\frac{1}{x}-1} dx;$ | 2) $\int \frac{(5x+6)dx}{x^2+4x-12};$ | 3) $\int x \sin 5x dx.$ |
| 7. 1) $\int \frac{dx}{7+4x^2};$ | 2) $\int \frac{(5x-7)dx}{x^2-x-20};$ | 3) $\int (2x+5) \sin x dx.$ |
| 8. 1) $\int \frac{dx}{\cos^3 2x};$ | 2) $\int \frac{5x dx}{x^2+x-6};$ | 3) $\int \frac{\ln x dx}{\sqrt{x}}.$ |
| 9. 1) $\int \cos\left(\frac{x}{3}-4\right) dx;$ | 2) $\int \frac{(5x+7)dx}{x^2+2x-8};$ | 3) $\int \arcsin \frac{x}{3} dx.$ |
| 10. 1) $\int \frac{dx}{\sqrt{(2x+1)^2}};$ | 2) $\int \frac{(5x+1)dx}{x^2+2x-15};$ | 3) $\int xe^{3x} dx.$ |
| 11. 1) $\int \frac{e^x dx}{\sqrt{1-e^x}};$ | 2) $\int \frac{19-4x}{2x^2+x-3} dx;$ | 3) $\int (5x-2) \ln x dx.$ |
| 12. 1) $\int x\sqrt{3-x^2} dx;$ | 2) $\int \frac{2x+9}{x^2+5x+6} dx;$ | 3) $\int x \cdot \cos^3 2x dx.$ |
| 13. 1) $\int \frac{\operatorname{arctg} x dx}{1+x^2};$ | 2) $\int \frac{x+9}{x^2+2x-3} dx;$ | 3) $\int \ln(3+x^2) dx.$ |
| 14. 1) $\int \sin 2x \sqrt{2-\cos^2 x} dx;$ | 2) $\int \frac{2x+27}{x^2-x-12} dx;$ | 3) $\int x \cdot \arcsin x dx.$ |
| 15. 1) $\int \frac{\sin x dx}{1-\cos x};$ | 2) $\int \frac{4x+31}{2x^2+11x+12} dx;$ | 3) $\int (2-x) \sin x dx.$ |
| 16. 1) $\int \frac{\sqrt{\ln x} dx}{x};$ | 2) $\int \frac{11x-2}{x^2+x-2} dx;$ | 3) $\int (1-\ln x) dx.$ |
| 17. 1) $\int \frac{1-\lg x}{\cos^2 x} dx;$ | 2) $\int \frac{17-2x}{x^2-5x+4} dx;$ | 3) $\int (3x+4) \cos x dx.$ |
| 18. 1) $\int \frac{x^2 dx}{8+x^2};$ | 2) $\int \frac{9-2x}{x^2-5x+6} dx;$ | 3) $\int \operatorname{arccotg}(4x) dx.$ |
| 19. 1) $\int \frac{\sin 2x dx}{\cos^2 x+3};$ | 2) $\int \frac{4x+27}{2x^2-x-6} dx;$ | 3) $\int x \ln^2 x dx.$ |
| 20. 1) $\int \frac{x^2 dx}{\cos^2 x^2};$ | 2) $\int \frac{x-13}{x^2-2x-8} dx;$ | 3) $\int x^2 \sin 3x dx.$ |

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II. Вычислить определенный интеграл.

1. $\int_{-1}^1 \frac{\sqrt{x+2} dx}{x};$

2. $\int_{-2}^0 \frac{3x dx}{\sqrt{(x+1)^3}};$

3. $\int_{-4}^0 \frac{\sqrt{x} dx}{4-x};$

4. $\int_{-4}^0 \frac{dx}{5-\sqrt{x^2}};$

5. $\int_{-4}^0 \frac{dx}{\sqrt{x-3}};$

6. $\int_{-4}^0 \frac{x dx}{\sqrt{(5-x)^3}};$

7. $\int_{-2}^0 \frac{dx}{2-\sqrt{1+x}};$

8. $\int_{-1}^0 \frac{dx}{8+\sqrt{x^2}};$

9. $\int_{-2}^0 \frac{dx}{1+\sqrt{3x+1}};$

10. $\int_{-1}^0 \frac{dx}{4+\sqrt{x^2}};$

11. $\int_{-4}^0 \frac{\sqrt{x} dx}{4+x};$

12. $\int_{-4}^0 \frac{\sqrt{x-3} dx}{x};$

13. $\int_{-1}^0 \frac{x^2+\sqrt{1+x}}{\sqrt{1+x}} dx.$

14. $\int_{-1}^0 \frac{\sqrt{x-2} dx}{1+\sqrt{x-2}};$

15. $\int_{-4}^0 \frac{\sqrt{x} dx}{x-6};$

16. $\int_{-4}^0 \frac{x dx}{\sqrt{2x+7}};$

17. $\int_{-4}^0 \frac{\sqrt{x^2}}{\sqrt{x^2+3}} dx.$

18. $\int_{-4}^0 \frac{dx}{\sqrt{x(x-1)}};$

19. $\int_{-4}^0 \frac{dx}{2+\sqrt{x-1}};$

20. $\int_{-4}^0 \frac{dx}{\sqrt{x+5}};$

III. Вычислить несобственный интеграл или установить его расходимость.

1. $\int_{-1}^1 \frac{dx}{\sqrt{x+3}};$

2. $\int_0^{\infty} xe^{-x^2} dx.$

3. $\int_{-1}^1 \frac{dx}{x \ln^2 x};$

4. $\int_{-1}^1 \frac{dx}{(x-4)^2};$

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$$3. \int_1^2 \frac{x dx}{x-1}$$

$$7. \int_0^{\pi/4} \frac{\cos x dx}{\sin^2 x}$$

$$9. \int_{-1}^0 \frac{dx}{\sqrt{(x+1)^3}}$$

$$11. \int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

$$13. \int_0^1 \frac{3x+2}{\sqrt{x}} dx$$

$$15. \int_0^e \frac{dx}{x \ln^2 x}$$

$$17. \int_{-1}^0 \frac{dx}{\sqrt{(x+1)^3}}$$

$$19. \int_{-5}^{-4} \frac{dx}{\sqrt{(x+5)^2}}$$

$$6. \int_{-27}^0 \lg x dx$$

$$8. \int_0^1 \frac{dx}{x^2+2x+5}$$

$$10. \int_0^{\pi/2} \operatorname{ctg} x dx$$

$$12. \int_0^1 \frac{dx}{9x^2+1}$$

$$14. \int_0^1 e^{-3x} dx$$

$$16. \int_{-1}^0 e^{-x^2} x dx$$

$$18. \int_0^1 \frac{dx}{4x^2+1}$$

$$20. \int_{16}^2 \frac{dx}{\sqrt{x+2}}$$

IV. Вычислить площадь плоской фигуры, ограниченной заданными кривыми. Силить чертёж области.

- $3x^2 - 4y = 0, 2x - 4y + 1 = 0.$
- $3x^2 + 4y = 0, 2x - 4y - 1 = 0.$
- $2x + 3y^2 = 0, 2x + 2y + 1 = 0.$
- $3x^2 - 4y = 0, 2x + 4y - 1 = 0.$
- $3x^2 + 4y = 0, 2x + 4y + 1 = 0.$
- $2x - 3y^2 = 0, 2x + 2y - 1 = 0.$
- $3x^2 - 2y = 0, 2x - 2y + 1 = 0.$
- $4x + 3y^2 = 0, 4x + 2y + 1 = 0.$
- $3x^2 - 2y = 0, 2x + 2y - 1 = 0.$
- $4x - 3y^2 = 0, 4x + 2y - 1 = 0.$
- $y = x^2 + 3, x = 0, y = x - 1, x = 2.$
- $y = x^2 + 2, x = 0, y = x - 2, x = 2.$
- $y = x^2 + 1, x = 0, y = x - 3, x = 2.$

- $y = x^3 - 1, x = 0, y = x - 5, x = 2.$
- $y = x^3 - 2, x = 0, y = x - 6, x = 2.$
- $y = x^3 + 3, x = 0, y = x + 7, x = -2.$
- $y = x^3 + 2, x = 0, y = x + 6, x = -2.$
- $y = x^3 + 1, x = 0, y = x + 5, x = -2.$
- $y = x^3 - 1, x = 0, y = x + 3, x = -2.$
- $y = x^3 - 2, x = 0, y = x + 2, x = -3.$

V. Вычислить объем тела, образованного вращением вокруг оси Oх кривой L.

- $x^2 - y = 0, x = -1, y = 0.$
- $x^2 + y = 0, x = 0, y = -1.$
- $x^2 + 2 = 0, x = 1, y = 0.$
- $x^2 - y = 0, x = 0, y = 1.$
- $x^2 - y = 0, x = 1, y = 0.$
- $x - y^2 = 0, x = 1, y = 0.$
- $x - y^2 = 0, x = 0, y = -1.$
- $x + y^2 = 0, x = -1, y = 0.$
- $x - y^2 = 0, x = 0, y = 1.$
- $x + y^2 = 0, x = 0, y = 1.$
- $x = -4x^2, x = 0, y = 4.$
- $y = 4x^2, x = 0, y = 4.$
- $y = 1 + 8x^2, x = 0, y = 9.$
- $y = -4x^2, x = -1, y = 0.$
- $y = 4x^2, x = -1, y = 0.$
- $x^2 + y = 0, x = 0, y = -1.$
- $x^2 - y = 0, x = 0, y = 1.$
- $x - y^2 = 0, x = 1, y = 0.$
- $x + y^2 = 0, x = -1, y = 0.$
- $y = -4x^2, x = 1, y = 0.$
- $y = 4x^2, x = 1, y = 0.$
- $y = 4x^2, x = 0, y = -4.$
- $y = -4x^2, x = 0, y = -4.$
- $y = 1 + 8x^2, x = -1/2, y = 1.$

КОНТРОЛЬНАЯ РАБОТА № 5

I. Изменить порядок интегрирования в двойном интеграле. Силить чертёж области интегрирования.

$$1. \int_{-1}^0 dx \int_{-2x+4}^{-2x^2} f(x, y) dy$$

$$3. \int_0^1 dx \int_{2x^2}^{4x+4} f(x, y) dy$$

$$5. \int_{-1}^0 dx \int_{4x-4}^{2x^2} f(x, y) dy$$

$$7. \int_0^1 dx \int_{-2x-4}^{-2x^2} f(x, y) dy$$

$$9. \int_0^1 dx \int_{-4x-4}^{-2x^2} f(x, y) dy$$

$$11. \int_0^1 dx \int_0^{\sqrt{4x-x^2}} f(x, y) dy$$

$$13. \int_0^1 dx \int_{-\sqrt{4x-x^2}}^0 f(x, y) dy$$

$$2. \int_0^1 dy \int_{-4y-4}^{-2y^2} f(x, y) dx$$

$$4. \int_{-1}^0 dy \int_{2y-4}^{4y^2} f(x, y) dx$$

$$6. \int_0^1 dy \int_{4y^2}^{2y+4} f(x, y) dx$$

$$8. \int_{-1}^0 dy \int_{-2y^2}^{-2y+4} f(x, y) dx$$

$$10. \int_{-1}^0 dy \int_{4y-4}^{2y^2} f(x, y) dx$$

$$12. \int_{-1}^0 dy \int_0^{\sqrt{-4y-y^2}} f(x, y) dx$$

$$14. \int_{-1}^0 dy \int_{-\sqrt{-4y-y^2}}^0 f(x, y) dx$$