

Задача 9. Тройной интеграл $\iiint_T f(x, y, z) dx dy dz$ за повторного, если область ограничена поверхностями.

- 9.1. $z = xy, x + 5y = 5, x \geq 0, y \geq 0, z \geq 0.$
- 9.2. $z = 16 - x^2 - y^2, x + y = 4, x \geq 0, y \geq 0, z \geq 0.$
- 9.3. $z = 3 + x^2 + y^2, 3x + 2y = 6, x \geq 0, y \geq 0, z \geq 0.$
- 9.4. $z = 4\sqrt{y}, y = 2x, x = 3, y \geq 0, z \geq 0.$
- 9.5. $z^2 = x^2 + y^2, y = x + 1, y = 1 - x, y = 0, z \geq 0.$
- 9.6. $z^2 = 4 - x^2 - y^2, y = 2x, y = 2, x \geq 0, z \geq 0.$
- 9.7. $z^2 = 4(x^2 + y^2), y = 3x, x = 2, y \geq 0, z \geq 0.$
- 9.8. $z = 9 - x^2 - y^2, y = x, y = 1, x \geq 0, z \geq 0.$
- 9.9. $z^2 = 9 - x^2 - y^2, x + y = 3, x \geq 0, y \geq 0, z \geq 0.$
- 9.10. $z = 3x^2 + 2y^2, x + y = 1, x \geq 0, y \geq 0, z \geq 0.$

- 9.11. $z = xy, 3x + 4y = 12, x \geq 0, y \geq 0, z \geq 0.$
- 9.12. $z = 3x^2, x = 3y, x = 3, y = 3x, z \geq 0.$
- 9.13. $z^2 = x^2 + y^2, y = 5x, y = 10, x \geq 0, z \geq 0.$
- 9.14. $y = 2\sqrt{z}, y = 4x, x = 2, z \geq 0.$
- 9.15. $z = x^2 + 4y^2, x = y, y = -2x, y = 10, z \geq 0.$
- 9.16. $z^2 = 4 - x^2 - y^2, x + y = 2, y - x = 2, y \geq 0, z \geq 0.$
- 9.17. $z = 2x^2, y = 3x, x = 1, y = 2x, z \geq 0.$
- 9.18. $z = 5y^2, x = 5y, x = 5, y \geq 0, z \geq 0.$
- 9.19. $z^2 = 3(x^2 + y^2), y = x, y = -x, y = 2, z \geq 0.$
- 9.20. $x + y + z = 3, x = 4y, y = 0, x = 4, z \geq 0.$
- 9.21. $z = 4y^2, x = 4y, x = 4, y = 0, z \geq 0.$
- 9.22. $z = 2x^2 + y^2, y = x, y = 5, x = 0, z \geq 0.$
- 9.23. $z = 2\sqrt{x}, y = 2x, y = 2, z \geq 0.$
- 9.24. $z = 2(x^2 + y^2), y = 3x, y = 3, x \geq 0, z \geq 0.$
- 9.25. $z = \sqrt{3y}, y = 4x, x = 1, z \geq 0.$
- 9.26. $z = 3\sqrt{x}, y = 4x, x = 2, z \geq 0.$
- 9.27. $z^2 = 2(x^2 + y^2), y = 3x, x = 1, y = -3x, z \geq 0.$
- 9.28. $z = 3x^2, y = x, x = 3y, y = -\sqrt{x}, z \geq 0.$
- 9.29. $x = 3\sqrt{z}, y = 3x, y = 3, x \geq 0, z \geq 0.$
- 9.30. $z = y^2, y = 4x, y = 8, x \geq 0, z \geq 0.$

Задача 10. Вычислить тройной интеграл $\iiint_G f(x, y, z) dv$ по области G , ограниченной следующими поверхностями.

- 10.1. $\iiint_G y dv, G : x + y + z = 4, x = 0, y = 0, z = 0.$
- 10.2. $\iiint_G \frac{dv}{(1 + x + y + z)^3}, G : x + y + z = 1, x = 0, y = 0, z = 0.$
- 10.3. $\iiint_G x dv, G : x + y = 1, x - y = 1, x + z = 1, x = 0, z = 0.$

- 10.4. $\iiint_G z \, dv, G: z = \sqrt{1-x^2}, x = |y|, z = 0.$
- 10.5. $\iiint_G xyz \, dv, G: y = x^2, x = y^2, z = xy, z = 0.$
- 10.6. $\iiint_G xy^2z^3 \, dv, G: y = x, x = 1, z = xy, z = 0.$
- 10.7. $\iiint_G x^2y^2 \, dv, G: y = x, x = 0, z = y, z = 1.$
- 10.8. $\iiint_G z^3 \, dv, G: z = y + x, x = y, x = 1, y = 0, z = 0.$
- 10.9. $\iiint_G y \, dv, G: z = y^2 - x^2, y = 1, z = 0.$
- 10.10. $\iiint_G z \, dv, G: z^2 = x^2 + y^2, y = x, x = 1, y = 0, z = 0 (z \geq 0).$
- 10.11. $\iiint_G x^2 \, dv, G: x + y + z = 2, x = 0, y = 0, z = 0.$
- 10.12. $\iiint_G \frac{dv}{z}, G: x = \sqrt{1-y^2}, x = 0, z = e^{2x}, z = e^x.$
- 10.13. $\iiint_G \sqrt{y} \, dv, G: z = \sqrt{y}, y = \sqrt{x}, z = 0, y = 2.$
- 10.14. $\iiint_G \frac{dv}{\left(1 + \frac{x}{2} + \frac{y}{3} + \frac{z}{4}\right)^4}, G: 6x + 4y + 3z = 12, x = 0, y = 0, z = 0.$
- 10.15. $\iiint_G y \, dv, G: z = y^2 + x^2, x + y = 3, x = 0, y = 0, z = 0.$
- 10.16. $\iiint_G (x^2 + 1) \, dv, G: z = xy, x + y = 1, x = 0, y = 0, z = 0.$
- 10.17. $\iiint_G x \, dv, G: 2x + 2y + z = 6, x = 0, y = 0, z = 0.$
- 10.18. $\iiint_G e^z \, dv, G: z = \ln(5-x), x + y = 2, x - y = 2, x = 0, z = 0.$
- 10.19. $\iiint_G x \, dv, G: z = y^2, 2x + 3y = 12, x = 0, y = 0, z = 0.$

- 11.5. $\iiint_G y^2 dv, G: x^2 + y^2 \leq 25, z^2 \leq x^2 + y^2, y \geq \frac{\sqrt{3}}{3}x, x \geq 0, z \geq 0.$
- 11.6. $\iiint_G z dv, G: x^2 + y^2 + 2x \leq 0, y \geq -x, z = 5, z \geq 0.$
- 11.7. $\iiint_G \frac{y^2 z dv}{\sqrt{(x^2 + y^2)^3}}, G: z = 9(x^2 + y^2), z = 9, y \leq x, y \geq 0.$
- 11.8. $\iiint_G z dv, G: z = \sqrt{x^2 + y^2}, z \geq 0, z = 4, y \leq \sqrt{3}x, y \geq 0.$
- 11.9. $\iiint_G \frac{dv}{\sqrt{x^2 + y^2}}, G: z \leq y, x^2 + y^2 \leq 1, z \geq 0, y \leq \sqrt{3}x, y \geq 0.$
- 11.10. $\iiint_G y dv, G: z \leq x, x^2 + y^2 \leq 2x, z \geq 0, y \leq x, y \geq 0.$
- 11.11. $\iiint_G \frac{xz dv}{\sqrt{x^2 + y^2}}, G: z \geq 2(x^2 + y^2), z \leq 2, y \leq x/\sqrt{3}, y \geq 0.$
- 11.12. $\iiint_G \frac{y dv}{\sqrt{x^2 + y^2}}, G: x^2 + y^2 \leq 4x, x + z \leq 4, y \geq -x/\sqrt{3}, y \leq 0, z \geq 0.$
- 11.13. $\iiint_G \frac{x dv}{\sqrt{x^2 + y^2}}, G: x^2 + y^2 \leq 8y, y + z \leq 8, y \geq \sqrt{3}x, x \geq 0, z \geq 0.$
- 11.14. $\iiint_G \frac{xy dv}{\sqrt{(x^2 + y^2)^3}}, G: z = x^2 + y^2, z = 16, y \geq x, x \geq 0.$
- 11.15. $\iiint_G \frac{x dv}{\sqrt{x^2 + y^2}}, G: x^2 + y^2 \leq 16y, y - z \leq 16, y \geq \sqrt{3}x, x \geq 0.$
- 11.16. $\iiint_G \frac{dv}{\sqrt{x^2 + y^2}}, G: x^2 + y^2 \leq 2x, x - z \leq 4, y \leq \frac{x}{\sqrt{3}}, y \geq 0, z \leq 0.$
- 11.17. $\iiint_G \frac{e^{\sqrt{x^2 + y^2}} dv}{\sqrt{x^2 + y^2}}, G: z \leq y, x^2 + y^2 \leq 4, y \geq |x|, z \geq 0.$
- 11.18. $\iiint_G \frac{x dv}{\sqrt{x^2 + y^2}}, G: x^2 + y^2 \geq 2x, x^2 + y^2 \leq 4x, z = 4, y \geq 0, z \geq 0.$
- 11.19. $\iiint_G \frac{x dv}{x^2 + y^2}, G: z \leq x, x^2 + y^2 \leq 16, z \geq 0, y \leq \frac{x}{\sqrt{3}}, y \geq 0.$
- 11.20. $\iiint_G \frac{y dv}{x^2 + y^2}, G: z \leq x^2, x^2 + y^2 \leq 9, x \geq |\sqrt{3}y|, z \geq 0.$

- 12.21. $\iiint_G \frac{(x-y)dv}{\sqrt{x^2+y^2+z^2}}, G: 1 \leq x^2+y^2+z^2 \leq 4, x \leq 0, y \geq 0, z \geq \sqrt{x^2+y^2}.$
- 12.22. $\iiint_G ydv, G: z \leq \sqrt{4-x^2-y^2}, z \geq \sqrt{x^2+y^2}, x \leq 0, y \geq -x.$
- 12.23. $\iiint_G \frac{dv}{z^2}, G: z^2 \geq 3(x^2+y^2), 1 \leq x^2+y^2+z^2 \leq 4, x \leq 0, y \geq 0, z \geq 0.$
- 12.24. $\iiint_G \frac{dv}{\sqrt{x^2+y^2+z^2}}, G: 4 \leq x^2+y^2+z^2 \leq 16, x \geq 0, y \geq 0, z \leq \sqrt{x^2+y^2}.$
- 12.25. $\iiint_G \frac{z^2 dv}{\sqrt{(x^2+y^2+z^2)^5}}, G: \begin{cases} 1 \leq x^2+y^2+z^2 \leq 4, z \geq \sqrt{3(x^2+y^2)}, \\ y \leq x, y \geq 0. \end{cases}$
- 12.26. $\iiint_G \frac{ydv}{\sqrt{x^2+y^2+z^2}}, G: 9 \leq x^2+y^2+z^2 \leq 25, y \leq -\frac{x}{\sqrt{3}}, y \geq 0, z \geq 0.$
- 12.27. $\iiint_G \frac{(x^2-y^2)dv}{(x^2+y^2+z^2)^2}, G: x^2+y^2+z^2 \leq 25, z^2 \leq x^2+y^2, x \geq 0, y \geq x.$
- 12.28. $\iiint_G zdv, G: x^2+y^2+z^2 \leq 4, z^2 \leq x^2+y^2 \leq 4, z \geq 0, y \geq |\sqrt{3}x|.$
- 12.29. $\iiint_G \frac{y^2 dv}{(x^2+y^2+z^2)^2}, G: 16 \leq x^2+y^2+z^2 \leq 36, y \geq \left| \frac{x}{\sqrt{3}} \right|, z \geq 0.$
- 12.30. $\iiint_G \frac{zdv}{x^2+y^2+z^2}, G: 4 \leq x^2+y^2+z^2 \leq 16, z \geq \sqrt{x^2+y^2}, y \geq |x|.$

Задача 13. Найти объем тела, заданного ограничивающими его поверхностями.

- 13.1. $x^2+y^2=2y, z=\frac{9}{4}-x^2, z=0.$
- 13.2. $x^2+y^2=2x, z=\frac{17}{4}-y^2, z=0.$
- 13.3. $x^2+y^2=2y, x^2+y^2=5y, z=\sqrt{x^2+y^2}, z=0.$
- 13.4. $x^2+y^2=9x, x^2+y^2=12x, z=\sqrt{x^2+y^2}, z=0, y=0 (y \geq 0).$
- 13.5. $x^2+y^2=-2\sqrt{2}y, z=x^2+y^2-4, z=0 (z \geq 0).$
- 13.6. $x^2+y^2=4x, z=10-y^2, z=0.$
- 13.7. $x^2+y^2=-2\sqrt{2}x, z=x^2+y^2-4, z=0 (z \geq 0)$

- 13.8. $x^2 + y^2 = 4y, z = 6 - x^2, z = 0.$
- 13.9. $x^2 + y^2 = 7x, x^2 + y^2 = 10x, z = \sqrt{x^2 + y^2}, z = 0$
- 13.10. $x^2 + y^2 = 2\sqrt{2}x, z = x^2 + y^2 - 4, z = 0 (z \geq 0).$
- 13.11. $x^2 + y^2 = 10x, x^2 + y^2 = 13x, z = \sqrt{x^2 + y^2}, z = 0$
- 13.12. $x^2 + y^2 = 8\sqrt{2}y, z = x^2 + y^2 - 64, z = 0 (z \geq 0).$
- 13.13. $x^2 + y^2 = 2y, z = \frac{13}{4} - x^2, z = 0.$
- 13.14. $x^2 + y^2 = 3y, x^2 + y^2 = 6y, z = \sqrt{x^2 + y^2}, z = 0.$
- 13.15. $x^2 + y^2 = 2x, z = \frac{21}{4} - y^2, z = 0.$
- 13.16. $x^2 + y^2 = 5y, x^2 + y^2 = 8y, z = \sqrt{x^2 + y^2}, z = 0.$
- 13.17. $x^2 + y^2 = 6\sqrt{2}x, z = x^2 + y^2 - 36, z = 0 (z \geq 0).$
- 13.18. $x^2 + y^2 = 2\sqrt{2}y, z = x^2 + y^2 - 4, z = 0 (z \geq 0).$
- 13.19. $x^2 + y^2 = 4x, z = 12 - y^2, z = 0.$
- 13.20. $x^2 + y^2 = 8x, x^2 + y^2 = 11x, z = \sqrt{x^2 + y^2}, z = 0.$
- 13.21. $x^2 + y^2 = 4y, x^2 + y^2 = 7y, z = \sqrt{x^2 + y^2}, z = 0.$
- 13.22. $x^2 + y^2 = 4\sqrt{2}y, z = x^2 + y^2 - 16, z = 0 (z \geq 0).$
- 13.23. $x^2 + y^2 = 4y, z = 4 - x^2, z = 0.$
- 13.24. $x^2 + y^2 = 4\sqrt{2}x, z = x^2 + y^2 - 16, z = 0 (z \geq 0).$
- 13.25. $x^2 + y^2 = 2y, z = \frac{5}{4} - x^2, z = 0.$
- 13.26. $x^2 + y^2 = 8\sqrt{2}x, z = x^2 + y^2 - 64, z = 0 (z \geq 0).$
- 13.27. $x^2 + y^2 = 6x, x^2 + y^2 = 9x, z = \sqrt{x^2 + y^2}, z = 0, y = 0 (y \leq 0).$
- 13.28. $x^2 + y^2 = -4x, z = 8 - y^2, z = 0.$
- 13.29. $x^2 + y^2 = 6\sqrt{2}y, z = x^2 + y^2 - 36, z = 0 (z \geq 0).$
- 13.30. $x^2 + y^2 = y, x^2 + y^2 = 4y, z = \sqrt{x^2 + y^2}, z = 0.$

Задача 14. Найти координаты центра масс тела, заданного ограничивающими его поверхностями.

14.1. $z = \sqrt{9 - x^2 - y^2}, 3(z + 1) = \sqrt{x^2 + y^2}.$