ВАРИАНТ 1

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 9\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{3x}{40}, & 0 \le x < 4, \\ \dfrac{3(8-x)}{40}, & 4 \le x \le 8 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(10-x)}{120}, \quad 0 \le x \le 10$$

3. $\dfrac{\partial u}{\partial t} = 4\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{x}{20}, & 0 \le x < 10, \\ \dfrac{20-x}{20}, & 10 \le x \le 20 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \{(x,y) : 0 \le x \le 2; \quad 0 \le y \le 8\}$

ВАРИАНТ 2

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 9\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \dfrac{3x}{40}, & 0 \le x < 4, \\ \dfrac{3(8-x)}{40}, & 4 \le x \le 8 \end{cases}$$

$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{x(10-x)}{120}, \quad 0 \le x \le 10$$

3. $\dfrac{\partial u}{\partial t} = 4\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \dfrac{x}{10}, & 0 \le x < 20, \\ \dfrac{40-x}{10}, & 20 \le x \le 40 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \left\{ (x,y): 0 \le x \le 2\pi; \quad 0 \le y \le \dfrac{\pi}{2} \right\}$

<u>ВАРИАНТ 3</u>

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 9\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{3x}{40}, & 0 \le x < 4, \\ \dfrac{3(8-x)}{40}, & 4 \le x \le 8 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(10-x)}{120}, \quad 0 \le x \le 10$$

3. $\dfrac{\partial u}{\partial t} = 8\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{x}{45}, & 0 \le x < 45, \\ \dfrac{90-x}{45}, & 45 \le x \le 90 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$$ в области Ω с границей s

Область $\Omega = \left\{ (x,y) : 0 \le x \le \dfrac{\pi}{3}; \quad 0 \le y \le 3\pi \right\}$

ВАРИАНТ 4

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 9\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{3x}{40}, & 0 \le x < 4, \\ \dfrac{3(8-x)}{40}, & 4 \le x \le 8 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(10-x)}{120}, \quad 0 \le x \le 10$$

3. $\dfrac{\partial u}{\partial t} = 36\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{x}{60}, & 0 \le x < 15, \\ \dfrac{30-x}{60}, & 15 \le x \le 30 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u|_s = 0$ в области Ω с границей s

Область $\Omega = \left\{ (x,y) : 0 \le x \le 3; \quad 0 \le y \le \dfrac{3}{4} \right\}$

ВАРИАНТ 5

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 9\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{3x}{40}, & 0 \le x < 4, \\ \dfrac{3(8-x)}{40}, & 4 \le x \le 8 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(10-x)}{120}, \quad 0 \le x \le 10$$

3. $\dfrac{\partial u}{\partial t} = 49\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{x}{56}, & 0 \le x < 35, \\ \dfrac{70-x}{56}, & 35 \le x \le 70 \end{cases}$$

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4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$$ в области Ω с границей s

Область $\Omega = \{(x,y): 0 \le x \le 9\pi; \quad 0 \le y \le 4\pi\}$

ВАРИАНТ 6

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 16\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \dfrac{x}{125}, & 0 \le x < 25, \\ \dfrac{3(50-x)}{125}, & 25 \le x \le 50 \end{cases}$$

$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = 49\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \le x \le 7$$

3. $\dfrac{\partial u}{\partial t} = 4\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \dfrac{x}{20}, & 0 \le x < 10, \\ \dfrac{20-x}{20}, & 10 \le x \le 20 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \{(x,y): 0 \le x \le 2; \quad 0 \le y \le 0{,}5\}$

ВАРИАНТ 7

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 16\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \dfrac{x}{125}, & 0 \le x < 25, \\ \dfrac{3(50-x)}{125}, & 25 \le x \le 50 \end{cases}$$

$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = 49\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \le x \le 7$$

3. $\dfrac{\partial u}{\partial t} = 4\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \dfrac{x}{10}, & 0 \le x < 20, \\ \dfrac{40-x}{10}, & 20 \le x \le 40 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \{(x,y): 0 \le x \le 2; \quad 0 \le y \le 8\}$

ВАРИАНТ 8

Решить краевые задачи

1. $\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{125}, & 0 \le x < 25, \\ \frac{3(50-x)}{125}, & 25 \le x \le 50 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\frac{\partial^2 u}{\partial t^2} = 49\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \le x \le 7$$

3. $\frac{\partial u}{\partial t} = 8\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{45}, & 0 \le x < 45, \\ \frac{90-x}{45}, & 45 \le x \le 90 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \left\{(x,y): 0 \le x \le 2\pi; \quad 0 \le y \le \frac{\pi}{2}\right\}$

ВАРИАНТ 9

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 16\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{x}{125}, & 0 \le x < 25, \\ \dfrac{3(50-x)}{125}, & 25 \le x \le 50 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = 49\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \le x \le 7$$

3. $\dfrac{\partial u}{\partial t} = 36\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{x}{60}, & 0 \le x < 15, \\ \dfrac{30-x}{60}, & 15 \le x \le 30 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \left\{ (x,y) : 0 \le x \le \dfrac{\pi}{3}; \quad 0 \le y \le 3\pi \right\}$

ВАРИАНТ 10

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 16\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{x}{125}, & 0 \le x < 25, \\ \dfrac{3(50-x)}{125}, & 25 \le x \le 50 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = 49\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \le x \le 7$$

3. $\dfrac{\partial u}{\partial t} = 49\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{x}{56}, & 0 \le x < 35, \\ \dfrac{70-x}{56}, & 35 \le x \le 70 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$$ в области Ω с границей s

Область $\Omega = \left\{(x,y): 0 \le x \le 3; \quad 0 \le y \le \dfrac{3}{4}\right\}$

ВАРИАНТ 11

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 12\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{x}{7}, & 0 \le x < 14, \\ \dfrac{(28 - x)}{7}, & 14 \le x \le 28 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = 49\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7 - x)}{343}, \quad 0 \le x \le 7$$

3. $\dfrac{\partial u}{\partial t} = 4\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{x}{20}, & 0 \le x < 10, \\ \dfrac{20 - x}{20}, & 10 \le x \le 20 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \{(x,y): 0 \le x \le 9\pi; \quad 0 \le y \le 4\pi\}$

ВАРИАНТ 12

Решить краевые задачи

1. $\frac{\partial^2 u}{\partial t^2} = 12\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{x}{7}, & 0 \le x < 14, \\ \frac{(28-x)}{7}, & 14 \le x \le 28 \end{cases}$$

$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\frac{\partial^2 u}{\partial t^2} = 49\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \le x \le 7$$

3. $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{x}{10}, & 0 \le x < 20, \\ \frac{40-x}{10}, & 20 \le x \le 40 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \{(x,y): 0 \le x \le 2; \quad 0 \le y \le 0,5\}$

ВАРИАНТ 13

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 12\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{x}{7}, & 0 \le x < 14, \\ \dfrac{(28-x)}{7}, & 14 \le x \le 28 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = 49\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \le x \le 7$$

3. $\dfrac{\partial u}{\partial t} = 8\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{x}{45}, & 0 \le x < 45, \\ \dfrac{90-x}{45}, & 45 \le x \le 90 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u|_s = 0$ в области Ω с границей s

Область $\Omega = \{(x,y): 0 \le x \le 2; \quad 0 \le y \le 8\}$

ВАРИАНТ 14

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 12\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{x}{7}, & 0 \le x < 14, \\ \dfrac{(28-x)}{7}, & 14 \le x \le 28 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = 49\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \le x \le 7$$

3. $\dfrac{\partial u}{\partial t} = 36\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{x}{60}, & 0 \le x < 15, \\ \dfrac{30-x}{60}, & 15 \le x \le 30 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \left\{ (x,y) : 0 \le x \le 2\pi; \quad 0 \le y \le \dfrac{\pi}{2} \right\}$

ВАРИАНТ 15

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 12\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{x}{7}, & 0 \le x < 14, \\ \dfrac{(28-x)}{7}, & 14 \le x \le 28 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = 49\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \le x \le 7$$

3. $\dfrac{\partial u}{\partial t} = 49\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{x}{56}, & 0 \le x < 35, \\ \dfrac{70-x}{56}, & 35 \le x \le 70 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \left\{ (x,y) : 0 \le x \le \dfrac{\pi}{3}; \quad 0 \le y \le 3\pi \right\}$

ВАРИАНТ 16

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 12\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \dfrac{x}{7}, & 0 \le x < 14, \\ \dfrac{(28-x)}{7}, & 14 \le x \le 28 \end{cases}$$

$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{x(10-x)}{120}, \quad 0 \le x \le 10$$

3. $\dfrac{\partial u}{\partial t} = 4\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \dfrac{x}{20}, & 0 \le x < 10, \\ \dfrac{20-x}{20}, & 10 \le x \le 20 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$$ в области Ω с границей s

Область $\Omega = \left\{(x,y): 0 \le x \le 3; \quad 0 \le y \le \dfrac{3}{4}\right\}$

ВАРИАНТ 17

Решить краевые задачи

1. $\frac{\partial^2 u}{\partial t^2} = 12\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{x}{7}, & 0 \le x < 14, \\ \frac{(28-x)}{7}, & 14 \le x \le 28 \end{cases}$$

$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{x(10-x)}{120}, \quad 0 \le x \le 10$$

3. $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{x}{10}, & 0 \le x < 20, \\ \frac{40-x}{10}, & 20 \le x \le 40 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \{(x,y): 0 \le x \le 9\pi; \quad 0 \le y \le 4\pi\}$

ВАРИАНТ 18

Решить краевые задачи

1. $\frac{\partial^2 u}{\partial t^2} = 12\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{7}, & 0 \le x < 14, \\ \frac{(28-x)}{7}, & 14 \le x \le 28 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(10-x)}{120}, \quad 0 \le x \le 10$$

3. $\frac{\partial u}{\partial t} = 8\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{45}, & 0 \le x < 45, \\ \frac{90-x}{45}, & 45 \le x \le 90 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \{(x,y): 0 \le x \le 2; \quad 0 \le y \le 0{,}5\}$

ВАРИАНТ 19

Решить краевые задачи

1. $\frac{\partial^2 u}{\partial t^2} = 12\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{x}{7}, & 0 \le x < 14, \\ \frac{(28-x)}{7}, & 14 \le x \le 28 \end{cases}$$

$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{x(10-x)}{120}, \quad 0 \le x \le 10$$

3. $\frac{\partial u}{\partial t} = 36\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{x}{60}, & 0 \le x < 15, \\ \frac{30-x}{60}, & 15 \le x \le 30 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$$ в области Ω с границей s

Область $\Omega = \left\{(x,y): 0 \le x \le 3; \quad 0 \le y \le \frac{3}{4}\right\}$

ВАРИАНТ 20

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2}=12\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t)=u(l,t)=0;$$

$$u(x,0)=\begin{cases}\dfrac{x}{7}, & 0\le x<14,\\ \dfrac{(28-x)}{7}, & 14\le x\le 28\end{cases}$$

$$\frac{\partial u}{\partial t}(x,0)=0$$

2. $\dfrac{\partial^2 u}{\partial t^2}=\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t)=u(l,t)=0;$$

$$u(x,0)=0$$

$$\frac{\partial u}{\partial t}(x,0)=\frac{x(10-x)}{120},\quad 0\le x\le 10$$

3. $\dfrac{\partial u}{\partial t}=49\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t)=u(l,t)=0;$$

$$u(x,0)=\begin{cases}\dfrac{x}{56}, & 0\le x<35,\\ \dfrac{70-x}{56}, & 35\le x\le 70\end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u=\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=\lambda u(x,y),\quad u\big|_s=0$ в области Ω с границей s

Область $\Omega=\{(x,y):0\le x\le 2;\quad 0\le y\le 8\}$

ВАРИАНТ 21

Решить краевые задачи

1. $\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{125}, & 0 \le x < 25, \\ \frac{3(50-x)}{125}, & 25 \le x \le 50 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(10-x)}{120}, \quad 0 \le x \le 10$$

3. $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{20}, & 0 \le x < 10, \\ \frac{20-x}{20}, & 10 \le x \le 20 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \left\{(x,y): 0 \le x \le 2\pi; \quad 0 \le y \le \frac{\pi}{2}\right\}$

ВАРИАНТ 22

Решить краевые задачи

1. $\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{125}, & 0 \le x < 25, \\ \frac{3(50-x)}{125}, & 25 \le x \le 50 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(10-x)}{120}, \quad 0 \le x \le 10$$

3. $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{10}, & 0 \le x < 20, \\ \frac{40-x}{10}, & 20 \le x \le 40 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \left\{(x,y): 0 \le x \le 3; \quad 0 \le y \le \frac{3}{4}\right\}$

ВАРИАНТ 23

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 16\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \dfrac{x}{125}, & 0 \le x < 25, \\ \dfrac{3(50-x)}{125}, & 25 \le x \le 50 \end{cases}$$

$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{x(10-x)}{120}, \quad 0 \le x \le 10$$

3. $\dfrac{\partial u}{\partial t} = 8\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \dfrac{x}{45}, & 0 \le x < 45, \\ \dfrac{90-x}{45}, & 45 \le x \le 90 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \{(x,y): 0 \le x \le 9\pi; \quad 0 \le y \le 4\pi\}$

ВАРИАНТ 24

Решить краевые задачи

1. $\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{x}{125}, & 0 \le x < 25, \\ \frac{3(50-x)}{125}, & 25 \le x \le 50 \end{cases}$$

$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{x(10-x)}{120}, \quad 0 \le x \le 10$$

3. $\frac{\partial u}{\partial t} = 36\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{x}{60}, & 0 \le x < 15, \\ \frac{30-x}{60}, & 15 \le x \le 30 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \{(x,y): 0 \le x \le 2; \quad 0 \le y \le 0{,}5\}$

ВАРИАНТ 25

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 16\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \dfrac{x}{125}, & 0 \le x < 25, \\ \dfrac{3(50-x)}{125}, & 25 \le x \le 50 \end{cases}$$

$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{x(10-x)}{120}, \quad 0 \le x \le 10$$

3. $\dfrac{\partial u}{\partial t} = 49\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \dfrac{x}{56}, & 0 \le x < 35, \\ \dfrac{70-x}{56}, & 35 \le x \le 70 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \left\{ (x,y): 0 \le x \le \dfrac{\pi}{3}; \quad 0 \le y \le 3\pi \right\}$

ВАРИАНТ 26

Решить краевые задачи

1. $\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{3x}{40}, & 0 \le x < 4, \\ \frac{3(8-x)}{40}, & 4 \le x \le 8 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\frac{\partial^2 u}{\partial t^2} = 49\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \le x \le 7$$

3. $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{20}, & 0 \le x < 10, \\ \frac{20-x}{20}, & 10 \le x \le 20 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \left\{(x,y): 0 \le x \le 3; \quad 0 \le y \le \frac{3}{4}\right\}$

ВАРИАНТ 27

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 9\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \dfrac{3x}{40}, & 0 \le x < 4, \\ \dfrac{3(8-x)}{40}, & 4 \le x \le 8 \end{cases}$$

$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = 49\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \le x \le 7$$

3. $\dfrac{\partial u}{\partial t} = 4\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \dfrac{x}{10}, & 0 \le x < 20, \\ \dfrac{40-x}{10}, & 20 \le x \le 40 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \{(x,y): 0 \le x \le 2; \quad 0 \le y \le 8\}$

ВАРИАНТ 28

Решить краевые задачи

1. $\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{3x}{40}, & 0 \le x < 4, \\ \frac{3(8-x)}{40}, & 4 \le x \le 8 \end{cases}$$

$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\frac{\partial^2 u}{\partial t^2} = 49\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \le x \le 7$$

3. $\frac{\partial u}{\partial t} = 8\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{x}{45}, & 0 \le x < 45, \\ \frac{90-x}{45}, & 45 \le x \le 90 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \left\{(x,y) : 0 \le x \le 2\pi; \quad 0 \le y \le \frac{\pi}{2}\right\}$

ВАРИАНТ 29

Решить краевые задачи

1. $\dfrac{\partial^2 u}{\partial t^2} = 9\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{3x}{40}, & 0 \le x < 4, \\ \dfrac{3(8-x)}{40}, & 4 \le x \le 8 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\dfrac{\partial^2 u}{\partial t^2} = 49\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \le x \le 7$$

3. $\dfrac{\partial u}{\partial t} = 36\dfrac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \dfrac{x}{60}, & 0 \le x < 15, \\ \dfrac{30-x}{60}, & 15 \le x \le 30 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u\big|_s = 0$ в области Ω с границей s

Область $\Omega = \{(x,y) : 0 \le x \le 9\pi; \quad 0 \le y \le 4\pi\}$

ВАРИАНТ 30

Решить краевые задачи

1. $\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{3x}{40}, & 0 \le x < 4, \\ \frac{3(8-x)}{40}, & 4 \le x \le 8 \end{cases}$$

$$\frac{\partial u}{\partial t}(x,0) = 0$$

2. $\frac{\partial^2 u}{\partial t^2} = 49\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \le x \le 7$$

3. $\frac{\partial u}{\partial t} = 49\frac{\partial^2 u}{\partial x^2}$;

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{x}{56}, & 0 \le x < 35, \\ \frac{70-x}{56}, & 35 \le x \le 70 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x,y), \quad u|_s = 0$ в области Ω с границей s

Область $\Omega = \{(x,y): 0 \le x \le 2; \quad 0 \le y \le 0{,}5\}$