

ВАРИАНТ 1

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 9 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0, t) = u(l, t) &= 0; \\ u(x, 0) &= \begin{cases} \frac{3x}{40}, & 0 \leq x < 4, \\ \frac{3(8-x)}{40}, & 4 \leq x \leq 8 \end{cases} \\ \frac{\partial u}{\partial t}(x, 0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0, t) = u(l, t) &= 0; \\ u(x, 0) &= 0 \\ \frac{\partial u}{\partial t}(x, 0) &= \frac{x(10-x)}{120}, \quad 0 \leq x \leq 10 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0, t) = u(l, t) &= 0; \\ u(x, 0) &= \begin{cases} \frac{x}{20}, & 0 \leq x < 10, \\ \frac{20-x}{20}, & 10 \leq x \leq 20 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \{(x, y): 0 \leq x \leq 2; \quad 0 \leq y \leq 8\}$$

ВАРИАНТ 2

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 9 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0, t) = u(l, t) &= 0; \\ u(x, 0) &= \begin{cases} \frac{3x}{40}, & 0 \leq x < 4, \\ \frac{3(8-x)}{40}, & 4 \leq x \leq 8 \end{cases} \\ \frac{\partial u}{\partial t}(x, 0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0, t) = u(l, t) &= 0; \\ u(x, 0) &= 0 \\ \frac{\partial u}{\partial t}(x, 0) &= \frac{x(10-x)}{120}, \quad 0 \leq x \leq 10 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0, t) = u(l, t) &= 0; \\ u(x, 0) &= \begin{cases} \frac{x}{10}, & 0 \leq x < 20, \\ \frac{40-x}{10}, & 20 \leq x \leq 40 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \left\{ (x, y) : 0 \leq x \leq 2\pi; \quad 0 \leq y \leq \frac{\pi}{2} \right\}$$

ВАРИАНТ 3

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 9 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{3x}{40}, & 0 \leq x < 4, \\ \frac{3(8-x)}{40}, & 4 \leq x \leq 8 \end{cases} \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= 0 \\ \frac{\partial u}{\partial t}(x,0) &= \frac{x(10-x)}{120}, \quad 0 \leq x \leq 10 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 8 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{45}, & 0 \leq x < 45, \\ \frac{90-x}{45}, & 45 \leq x \leq 90 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \left\{ (x, y) : 0 \leq x \leq \frac{\pi}{3}; \quad 0 \leq y \leq 3\pi \right\}$$

ВАРИАНТ 4

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 9 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0, t) = u(l, t) &= 0; \\ u(x, 0) &= \begin{cases} \frac{3x}{40}, & 0 \leq x < 4, \\ \frac{3(8-x)}{40}, & 4 \leq x \leq 8 \end{cases} \\ \frac{\partial u}{\partial t}(x, 0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0, t) = u(l, t) &= 0; \\ u(x, 0) &= 0 \\ \frac{\partial u}{\partial t}(x, 0) &= \frac{x(10-x)}{120}, \quad 0 \leq x \leq 10 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 36 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0, t) = u(l, t) &= 0; \\ u(x, 0) &= \begin{cases} \frac{x}{60}, & 0 \leq x < 15, \\ \frac{30-x}{60}, & 15 \leq x \leq 30 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \left\{ (x, y) : 0 \leq x \leq 3; \quad 0 \leq y \leq \frac{3}{4} \right\}$$

ВАРИАНТ 5

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 9 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) = u(l,t) &= 0; \\ u(x,0) &= \begin{cases} \frac{3x}{40}, & 0 \leq x < 4, \\ \frac{3(8-x)}{40}, & 4 \leq x \leq 8 \end{cases} \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) = u(l,t) &= 0; \\ u(x,0) &= 0 \\ \frac{\partial u}{\partial t}(x,0) &= \frac{x(10-x)}{120}, \quad 0 \leq x \leq 10 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 49 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) = u(l,t) &= 0; \\ u(x,0) &= \begin{cases} \frac{x}{56}, & 0 \leq x < 35, \\ \frac{70-x}{56}, & 35 \leq x \leq 70 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \{(x, y) : 0 \leq x \leq 9\pi; \quad 0 \leq y \leq 4\pi\}$$

ВАРИАНТ 6

Решить краевые задачи

$$1. \frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2};$$
$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{125}, & 0 \leq x < 25, \\ \frac{3(50-x)}{125}, & 25 \leq x \leq 50 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

$$2. \frac{\partial^2 u}{\partial t^2} = 49 \frac{\partial^2 u}{\partial x^2};$$
$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \leq x \leq 7$$

$$3. \frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2};$$
$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{20}, & 0 \leq x < 10, \\ \frac{20-x}{20}, & 10 \leq x \leq 20 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \{(x, y): 0 \leq x \leq 2; \quad 0 \leq y \leq 0,5\}$$

ВАРИАНТ 7

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2};$$
$$u(0, t) = u(l, t) = 0;$$
$$u(x, 0) = \begin{cases} \frac{x}{125}, & 0 \leq x < 25, \\ \frac{3(50-x)}{125}, & 25 \leq x \leq 50 \end{cases}$$
$$\frac{\partial u}{\partial t}(x, 0) = 0$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = 49 \frac{\partial^2 u}{\partial x^2};$$
$$u(0, t) = u(l, t) = 0;$$
$$u(x, 0) = 0$$
$$\frac{\partial u}{\partial t}(x, 0) = \frac{x(7-x)}{343}, \quad 0 \leq x \leq 7$$

$$3. \quad \frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2};$$
$$u(0, t) = u(l, t) = 0;$$
$$u(x, 0) = \begin{cases} \frac{x}{10}, & 0 \leq x < 20, \\ \frac{40-x}{10}, & 20 \leq x \leq 40 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \{(x, y): 0 \leq x \leq 2; \quad 0 \leq y \leq 8\}$$

ВАРИАНТ 8

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2};$$
$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{125}, & 0 \leq x < 25, \\ \frac{3(50-x)}{125}, & 25 \leq x \leq 50 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = 49 \frac{\partial^2 u}{\partial x^2};$$
$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \leq x \leq 7$$

$$3. \quad \frac{\partial u}{\partial t} = 8 \frac{\partial^2 u}{\partial x^2};$$
$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{45}, & 0 \leq x < 45, \\ \frac{90-x}{45}, & 45 \leq x \leq 90 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \left\{ (x, y) : 0 \leq x \leq 2\pi; \quad 0 \leq y \leq \frac{\pi}{2} \right\}$$

ВАРИАНТ 9

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) = u(l,t) &= 0; \\ u(x,0) &= \begin{cases} \frac{x}{125}, & 0 \leq x < 25, \\ \frac{3(50-x)}{125}, & 25 \leq x \leq 50 \end{cases} \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = 49 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) = u(l,t) &= 0; \\ u(x,0) &= 0 \\ \frac{\partial u}{\partial t}(x,0) &= \frac{x(7-x)}{343}, \quad 0 \leq x \leq 7 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 36 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) = u(l,t) &= 0; \\ u(x,0) &= \begin{cases} \frac{x}{60}, & 0 \leq x < 15, \\ \frac{30-x}{60}, & 15 \leq x \leq 30 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \left\{ (x, y) : 0 \leq x \leq \frac{\pi}{3}; \quad 0 \leq y \leq 3\pi \right\}$$

ВАРИАНТ 10

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{125}, & 0 \leq x < 25, \\ \frac{3(50-x)}{125}, & 25 \leq x \leq 50 \end{cases} \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = 49 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= 0 \\ \frac{\partial u}{\partial t}(x,0) &= \frac{x(7-x)}{343}, \quad 0 \leq x \leq 7 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 49 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{56}, & 0 \leq x < 35, \\ \frac{70-x}{56}, & 35 \leq x \leq 70 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \left\{ (x, y) : 0 \leq x \leq 3; \quad 0 \leq y \leq \frac{3}{4} \right\}$$

ВАРИАНТ 11

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 12 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{7}, & 0 \leq x < 14, \\ \frac{(28-x)}{7}, & 14 \leq x \leq 28 \end{cases} \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = 49 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= 0 \\ \frac{\partial u}{\partial t}(x,0) &= \frac{x(7-x)}{343}, \quad 0 \leq x \leq 7 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{20}, & 0 \leq x < 10, \\ \frac{20-x}{20}, & 10 \leq x \leq 20 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \{(x, y): 0 \leq x \leq 9\pi; \quad 0 \leq y \leq 4\pi\}$$

ВАРИАНТ 12

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 12 \frac{\partial^2 u}{\partial x^2};$$
$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{7}, & 0 \leq x < 14, \\ \frac{(28-x)}{7}, & 14 \leq x \leq 28 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = 49 \frac{\partial^2 u}{\partial x^2};$$
$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \leq x \leq 7$$

$$3. \quad \frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2};$$
$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{10}, & 0 \leq x < 20, \\ \frac{40-x}{10}, & 20 \leq x \leq 40 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \{(x, y): 0 \leq x \leq 2; \quad 0 \leq y \leq 0,5\}$$

ВАРИАНТ 13

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 12 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0, t) = u(l, t) &= 0; \\ u(x, 0) &= \begin{cases} \frac{x}{7}, & 0 \leq x < 14, \\ \frac{(28-x)}{7}, & 14 \leq x \leq 28 \end{cases} \\ \frac{\partial u}{\partial t}(x, 0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = 49 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0, t) = u(l, t) &= 0; \\ u(x, 0) &= 0 \\ \frac{\partial u}{\partial t}(x, 0) &= \frac{x(7-x)}{343}, \quad 0 \leq x \leq 7 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 8 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0, t) = u(l, t) &= 0; \\ u(x, 0) &= \begin{cases} \frac{x}{45}, & 0 \leq x < 45, \\ \frac{90-x}{45}, & 45 \leq x \leq 90 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \{(x, y): 0 \leq x \leq 2; \quad 0 \leq y \leq 8\}$$

ВАРИАНТ 14

Решить краевые задачи

$$1. \frac{\partial^2 u}{\partial t^2} = 12 \frac{\partial^2 u}{\partial x^2};$$

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{7}, & 0 \leq x < 14, \\ \frac{(28-x)}{7}, & 14 \leq x \leq 28 \end{cases}$$

$$\frac{\partial u}{\partial t}(x,0) = 0$$

$$2. \frac{\partial^2 u}{\partial t^2} = 49 \frac{\partial^2 u}{\partial x^2};$$

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \leq x \leq 7$$

$$3. \frac{\partial u}{\partial t} = 36 \frac{\partial^2 u}{\partial x^2};$$

$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{60}, & 0 \leq x < 15, \\ \frac{30-x}{60}, & 15 \leq x \leq 30 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \left\{ (x, y) : 0 \leq x \leq 2\pi; \quad 0 \leq y \leq \frac{\pi}{2} \right\}$$

ВАРИАНТ 15

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 12 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) = u(l,t) &= 0; \\ u(x,0) &= \begin{cases} \frac{x}{7}, & 0 \leq x < 14, \\ \frac{(28-x)}{7}, & 14 \leq x \leq 28 \end{cases} \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = 49 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) = u(l,t) &= 0; \\ u(x,0) &= 0 \\ \frac{\partial u}{\partial t}(x,0) &= \frac{x(7-x)}{343}, \quad 0 \leq x \leq 7 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 49 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) = u(l,t) &= 0; \\ u(x,0) &= \begin{cases} \frac{x}{56}, & 0 \leq x < 35, \\ \frac{70-x}{56}, & 35 \leq x \leq 70 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \left\{ (x, y) : 0 \leq x \leq \frac{\pi}{3}; \quad 0 \leq y \leq 3\pi \right\}$$

ВАРИАНТ 16

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 12 \frac{\partial^2 u}{\partial x^2}; \quad u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{x}{7}, & 0 \leq x < 14, \\ \frac{(28-x)}{7}, & 14 \leq x \leq 28 \end{cases}$$

$$\frac{\partial u}{\partial t}(x,0) = 0$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; \quad u(0,t) = u(l,t) = 0;$$

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{x(10-x)}{120}, \quad 0 \leq x \leq 10$$

$$3. \quad \frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}; \quad u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{x}{20}, & 0 \leq x < 10, \\ \frac{20-x}{20}, & 10 \leq x \leq 20 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \left\{ (x, y) : 0 \leq x \leq 3; \quad 0 \leq y \leq \frac{3}{4} \right\}$$

ВАРИАНТ 17

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 12 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{7}, & 0 \leq x < 14, \\ \frac{(28-x)}{7}, & 14 \leq x \leq 28 \end{cases} \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= 0 \\ \frac{\partial u}{\partial t}(x,0) &= \frac{x(10-x)}{120}, \quad 0 \leq x \leq 10 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{10}, & 0 \leq x < 20, \\ \frac{40-x}{10}, & 20 \leq x \leq 40 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \{(x, y): 0 \leq x \leq 9\pi; \quad 0 \leq y \leq 4\pi\}$$

ВАРИАНТ 18

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 12 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{7}, & 0 \leq x < 14, \\ \frac{(28-x)}{7}, & 14 \leq x \leq 28 \end{cases} \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= 0 \\ \frac{\partial u}{\partial t}(x,0) &= \frac{x(10-x)}{120}, \quad 0 \leq x \leq 10 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 8 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{45}, & 0 \leq x < 45, \\ \frac{90-x}{45}, & 45 \leq x \leq 90 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \{(x, y): 0 \leq x \leq 2; \quad 0 \leq y \leq 0,5\}$$

ВАРИАНТ 19

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 12 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{7}, & 0 \leq x < 14, \\ \frac{(28-x)}{7}, & 14 \leq x \leq 28 \end{cases} \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= 0 \\ \frac{\partial u}{\partial t}(x,0) &= \frac{x(10-x)}{120}, \quad 0 \leq x \leq 10 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 36 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{60}, & 0 \leq x < 15, \\ \frac{30-x}{60}, & 15 \leq x \leq 30 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \left\{ (x, y) : 0 \leq x \leq 3; \quad 0 \leq y \leq \frac{3}{4} \right\}$$

ВАРИАНТ 20

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 12 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) = u(l,t) &= 0; \\ u(x,0) &= \begin{cases} \frac{x}{7}, & 0 \leq x < 14, \\ \frac{(28-x)}{7}, & 14 \leq x \leq 28 \end{cases} \end{aligned}$$

$$\frac{\partial u}{\partial t}(x,0) = 0$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) = u(l,t) &= 0; \\ u(x,0) &= 0 \\ \frac{\partial u}{\partial t}(x,0) &= \frac{x(10-x)}{120}, \quad 0 \leq x \leq 10 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 49 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) = u(l,t) &= 0; \\ u(x,0) &= \begin{cases} \frac{x}{56}, & 0 \leq x < 35, \\ \frac{70-x}{56}, & 35 \leq x \leq 70 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \{(x, y): 0 \leq x \leq 2; \quad 0 \leq y \leq 8\}$$

ВАРИАНТ 21

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0, t) &= u(l, t) = 0; \\ u(x, 0) &= \begin{cases} \frac{x}{125}, & 0 \leq x < 25, \\ \frac{3(50-x)}{125}, & 25 \leq x \leq 50 \end{cases} \\ \frac{\partial u}{\partial t}(x, 0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0, t) &= u(l, t) = 0; \\ u(x, 0) &= 0 \\ \frac{\partial u}{\partial t}(x, 0) &= \frac{x(10-x)}{120}, \quad 0 \leq x \leq 10 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0, t) &= u(l, t) = 0; \\ u(x, 0) &= \begin{cases} \frac{x}{20}, & 0 \leq x < 10, \\ \frac{20-x}{20}, & 10 \leq x \leq 20 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \left\{ (x, y) : 0 \leq x \leq 2\pi; \quad 0 \leq y \leq \frac{\pi}{2} \right\}$$

ВАРИАНТ 22

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{125}, & 0 \leq x < 25, \\ \frac{3(50-x)}{125}, & 25 \leq x \leq 50 \end{cases} \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= 0 \\ \frac{\partial u}{\partial t}(x,0) &= \frac{x(10-x)}{120}, \quad 0 \leq x \leq 10 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{10}, & 0 \leq x < 20, \\ \frac{40-x}{10}, & 20 \leq x \leq 40 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \left\{ (x, y) : 0 \leq x \leq 3; \quad 0 \leq y \leq \frac{3}{4} \right\}$$

ВАРИАНТ 23

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{125}, & 0 \leq x < 25, \\ \frac{3(50-x)}{125}, & 25 \leq x \leq 50 \end{cases} \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= 0 \\ \frac{\partial u}{\partial t}(x,0) &= \frac{x(10-x)}{120}, \quad 0 \leq x \leq 10 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 8 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{45}, & 0 \leq x < 45, \\ \frac{90-x}{45}, & 45 \leq x \leq 90 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \{(x, y): 0 \leq x \leq 9\pi; \quad 0 \leq y \leq 4\pi\}$$

ВАРИАНТ 24

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2};$$
$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{125}, & 0 \leq x < 25, \\ \frac{3(50-x)}{125}, & 25 \leq x \leq 50 \end{cases}$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2};$$
$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(10-x)}{120}, \quad 0 \leq x \leq 10$$

$$3. \quad \frac{\partial u}{\partial t} = 36 \frac{\partial^2 u}{\partial x^2};$$
$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{60}, & 0 \leq x < 15, \\ \frac{30-x}{60}, & 15 \leq x \leq 30 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \{(x, y): 0 \leq x \leq 2; \quad 0 \leq y \leq 0,5\}$$

ВАРИАНТ 25

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{125}, & 0 \leq x < 25, \\ \frac{3(50-x)}{125}, & 25 \leq x \leq 50 \end{cases} \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= 0 \\ \frac{\partial u}{\partial t}(x,0) &= \frac{x(10-x)}{120}, \quad 0 \leq x \leq 10 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 49 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{56}, & 0 \leq x < 35, \\ \frac{70-x}{56}, & 35 \leq x \leq 70 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \left\{ (x, y) : 0 \leq x \leq \frac{\pi}{3}; \quad 0 \leq y \leq 3\pi \right\}$$

ВАРИАНТ 26

Решить краевые задачи

$$1. \frac{\partial^2 u}{\partial t^2} = 9 \frac{\partial^2 u}{\partial x^2}; \quad u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{3x}{40}, & 0 \leq x < 4, \\ \frac{3(8-x)}{40}, & 4 \leq x \leq 8 \end{cases}$$

$$2. \frac{\partial^2 u}{\partial t^2} = 49 \frac{\partial^2 u}{\partial x^2}; \quad \frac{\partial u}{\partial t}(x,0) = 0$$

$$u(0,t) = u(l,t) = 0;$$

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \leq x \leq 7$$

$$3. \frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}; \quad u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{x}{20}, & 0 \leq x < 10, \\ \frac{20-x}{20}, & 10 \leq x \leq 20 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \left\{ (x, y) : 0 \leq x \leq 3; \quad 0 \leq y \leq \frac{3}{4} \right\}$$

ВАРИАНТ 27

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 9 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{3x}{40}, & 0 \leq x < 4, \\ \frac{3(8-x)}{40}, & 4 \leq x \leq 8 \end{cases} \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = 49 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= 0 \\ \frac{\partial u}{\partial t}(x,0) &= \frac{x(7-x)}{343}, \quad 0 \leq x \leq 7 \end{aligned}$$

$$3. \quad \frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}; \quad \begin{aligned} u(0,t) &= u(l,t) = 0; \\ u(x,0) &= \begin{cases} \frac{x}{10}, & 0 \leq x < 20, \\ \frac{40-x}{10}, & 20 \leq x \leq 40 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \{(x, y): 0 \leq x \leq 2; \quad 0 \leq y \leq 8\}$$

ВАРИАНТ 28

Решить краевые задачи

$$1. \quad \frac{\partial^2 u}{\partial t^2} = 9 \frac{\partial^2 u}{\partial x^2}; \quad u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{3x}{40}, & 0 \leq x < 4, \\ \frac{3(8-x)}{40}, & 4 \leq x \leq 8 \end{cases}$$

$$2. \quad \frac{\partial^2 u}{\partial t^2} = 49 \frac{\partial^2 u}{\partial x^2}; \quad \frac{\partial u}{\partial t}(x,0) = 0$$
$$u(0,t) = u(l,t) = 0;$$
$$u(x,0) = 0$$
$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \leq x \leq 7$$

$$3. \quad \frac{\partial u}{\partial t} = 8 \frac{\partial^2 u}{\partial x^2}; \quad u(0,t) = u(l,t) = 0;$$
$$u(x,0) = \begin{cases} \frac{x}{45}, & 0 \leq x < 45, \\ \frac{90-x}{45}, & 45 \leq x \leq 90 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \left\{ (x, y) : 0 \leq x \leq 2\pi; \quad 0 \leq y \leq \frac{\pi}{2} \right\}$$

ВАРИАНТ 29

Решить краевые задачи

$$\begin{aligned} & u(0,t) = u(l,t) = 0; \\ 1. \quad & \frac{\partial^2 u}{\partial t^2} = 9 \frac{\partial^2 u}{\partial x^2}; \quad u(x,0) = \begin{cases} \frac{3x}{40}, & 0 \leq x < 4, \\ \frac{3(8-x)}{40}, & 4 \leq x \leq 8 \end{cases} \\ & \frac{\partial u}{\partial t}(x,0) = 0 \\ & u(0,t) = u(l,t) = 0; \\ 2. \quad & \frac{\partial^2 u}{\partial t^2} = 49 \frac{\partial^2 u}{\partial x^2}; \quad u(x,0) = 0 \\ & \frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \leq x \leq 7 \end{aligned}$$

$$\begin{aligned} & u(0,t) = u(l,t) = 0; \\ 3. \quad & \frac{\partial u}{\partial t} = 36 \frac{\partial^2 u}{\partial x^2}; \quad u(x,0) = \begin{cases} \frac{x}{60}, & 0 \leq x < 15, \\ \frac{30-x}{60}, & 15 \leq x \leq 30 \end{cases} \end{aligned}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \{(x, y): 0 \leq x \leq 9\pi; \quad 0 \leq y \leq 4\pi\}$$

ВАРИАНТ 30

Решить краевые задачи

$$1. \frac{\partial^2 u}{\partial t^2} = 9 \frac{\partial^2 u}{\partial x^2}; \quad u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{3x}{40}, & 0 \leq x < 4, \\ \frac{3(8-x)}{40}, & 4 \leq x \leq 8 \end{cases}$$

$$\frac{\partial u}{\partial t}(x,0) = 0$$

$$2. \frac{\partial^2 u}{\partial t^2} = 49 \frac{\partial^2 u}{\partial x^2}; \quad u(0,t) = u(l,t) = 0;$$

$$u(x,0) = 0$$

$$\frac{\partial u}{\partial t}(x,0) = \frac{x(7-x)}{343}, \quad 0 \leq x \leq 7$$

$$3. \frac{\partial u}{\partial t} = 49 \frac{\partial^2 u}{\partial x^2}; \quad u(0,t) = u(l,t) = 0;$$

$$u(x,0) = \begin{cases} \frac{x}{56}, & 0 \leq x < 35, \\ \frac{70-x}{56}, & 35 \leq x \leq 70 \end{cases}$$

4. Найти собственные значения и собственные функции задачи Дирихле

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \lambda u(x, y), \quad u|_s = 0 \text{ в области } \Omega \text{ с границей } s$$

$$\text{Область } \Omega = \{(x, y) : 0 \leq x \leq 2; \quad 0 \leq y \leq 0,5\}$$