

I. Плоская область D ограничена заданными линиями.

1) Сделайте схематический рисунок области D .

2) С помощью двойного интеграла найдите площадь области D .

1. $y = 2/(x^2 + 1), y = \sqrt{x}, x = -\sqrt{y}$. 2. $y = 2x - 6, x = 3 - \sqrt{y}, y = \log_2 x$.

3. $y = x^3, x + y = 2, y = 3x + 2$ при $x + y \leq 2$. 4. $y = \sqrt{x+1}, x - y = 1, x + y = -1$.

5. $y = 2^x, x = -\sqrt{2y}, x + 2y = 2$. 6. $y = \sqrt{x+2}, x = \sqrt{2y}, y = -x$.

7. $y = 0,5^x, y - x = 1, y + 4x = -4$. 8. $x = \sqrt{y} - 1, x = \sqrt{1-y}, y = x^2 - 1$.

9. $x = \sqrt{2-y}, y = -\sqrt{2x}, y = x$. 10. $y = \sqrt[3]{x}, x = -\sqrt{y+2}, y = 3x - 2$.

11. $y = -x^3, y - x = 2, x\sqrt{3} = \sqrt{2-y}$ при $y \geq -x^3$. 12. $y - x = 1, x + y = -1, x = \sqrt{y+1}$.

13. $y = 4^x, y = (x-1)^2, x = \frac{2}{\sqrt{y}}$. 14. $y = \sqrt{-x}, x = \sqrt{y}, y = \frac{2}{x^2+1}$.

15. $x + y = 1, y = 2^x, 4x - y = 4$. 16. $y = \sqrt{4-x}, x + y = -2,$
 $x = \sqrt{2y} - 2$.

17. $y = \sqrt{2-x}, x = -\sqrt{2y}, y = x$. 18. $x = \sqrt{2y}, y = 0,5^x, 2y - x = 2$.

19. $y = \sqrt[3]{x}, x + y = 2, y\sqrt{3} = -\sqrt{2-x}$ при $y \leq \sqrt[3]{x}$. 20. $x = \sqrt{1-y}, y - x = 1, y + x = -1$.

$$\begin{array}{ll}
23. 4y - x = 4, y = \log_2 x, x + y = 1. & 24. x = -\sqrt{y}, y = \sqrt{x}, y = 2 - x. \\
25. y = \log_2 x, y = -\sqrt{2x}, 2x + y = 2. & 26. x = \arcsin y, y = \frac{\pi}{2x}, y = \frac{8x}{\pi} \\
& \text{при } x \geq 0 \\
27. x = y^3, x = 1 - \sqrt{y+1}, x + y = 2. & 28. y = 2^x, x + y = 3, x + 3y = 3. \\
29. y = \sqrt{x}, x + y = 2, x - y = 2. & 30. y = 4^{-x}, y = (x+1)^2, x = -\frac{2}{\sqrt{y}}.
\end{array}$$

II. Тело T ограничено заданными поверхностями.

- 1) Сделайте схематический рисунок тела T .
- 2) С помощью тройного интеграла найдите объем тела T , перейдя к цилиндрическим или сферическим координатам.

1. $z = 2 + \sqrt{4 - x^2 - y^2}, z = \sqrt{x^2 + y^2}, x = 0$ при $x \leq 0$.
2. $z + 4 = x^2 + y^2, 9z = 5(x^2 + y^2), y = 0$ при $y \geq 0$.
3. $3z^2 = x^2 + y^2, z^2 = 3(x^2 + y^2), x^2 + y^2 + z^2 = 9$ при $z \geq 0$.
4. $x^2 + y^2 = z^2 - 5, \frac{z^2}{9} = \frac{x^2 + y^2}{4}, x = 0$ при $z \geq 0, x \geq 0$.
5. $\frac{z^2}{9} = 1 - \frac{x^2 + y^2}{25}, x^2 + y^2 = 9, z = 0, x = 0$ при $z \geq 0, x \leq 0$.
6. $z = 4 - 2\sqrt{x^2 + y^2}, z = \frac{x^2 + y^2}{2} - 2, y = 0$ при $y \geq 0$.
7. $z = \frac{5}{3}\sqrt{9 - x^2 - y^2}, z = \frac{5}{4}\sqrt{x^2 + y^2}, x = 0$ при $x \leq 0$.
8. $x^2 + y^2 + z^2 = 4, x^2 + y^2 + (z - 2)^2 = 4$ при $0 \leq z \leq 2$.
9. $z = \sqrt{4 - x^2 - y^2}, z = \sqrt{x^2 + y^2} - 2, x = 0$ при $x \geq 0$.
10. $z = 4 - x^2 - y^2, 9z = -5(x^2 + y^2), y = 0$ при $y \leq 0$.
11. $z = 2 - \sqrt{x^2 + y^2}, z = 4 - x^2 - y^2, x = 0$ при $x \leq 0$.

12. $z^2 = 3(x^2 + y^2)$, $x^2 + y^2 + z^2 = 2z$, $x = 0$ при $x \geq 0$,
 $0 \leq z \leq \sqrt{3(x^2 + y^2)}$.
13. $x^2 + y^2 + 4z^2 = 4$, $x^2 + y^2 = 1$, $x = 0$, $z = 0$ при $x^2 + y^2 \geq 1$,
 $x \leq 0$.
14. $2z = 4 - x^2 - y^2$, $z = 2\sqrt{x^2 + y^2} - 4$, $y = 0$ при $y \geq 0$, $z \geq -4$.
15. $z = \sqrt{x^2 + y^2}$, $z + 2 = x^2 + y^2$, $x = 0$ при $x \leq 0$.
16. $x^2 + y^2 + z^2 = 9$, $x^2 + y^2 + z^2 = 2\sqrt{3}z$, $x = 0$, $z = 0$ при $x \geq 0$,
 $0 \leq 2\sqrt{3}z \leq x^2 + y^2 + z^2$.
17. $z = -\sqrt{4 - x^2 - y^2}$, $z = 2 - \sqrt{x^2 + y^2}$, $y = 0$ при $y \geq 0$, $z \leq 2$.
18. $z = x^2 + y^2$, $z = \frac{5}{9}(x^2 + y^2) + 4$, $y = 0$ при $y \leq 0$.
19. $z = 2 - x^2 - y^2$, $z = -\sqrt{x^2 + y^2}$, $x = 0$ при $x \geq 0$.
20. $z^2 = x^2 + y^2$, $x^2 + y^2 + z^2 = 4z$, $x = 0$ при $x \leq 0$,
 $z \geq \sqrt{x^2 + y^2}$.
21. $4x^2 + 4y^2 + z^2 = 36$, $x^2 + y^2 = 4$, $y = 0$, $z = 0$ при
 $y \geq 0$, $z \leq 0$, $x^2 + y^2 \geq 4$.
22. $z = -2\sqrt{x^2 + y^2}$, $2(z + 6) = x^2 + y^2$, $y = 0$ при $y \leq 0$.
23. $z = \sqrt{x^2 + y^2 + 9}$, $4z = 5\sqrt{x^2 + y^2}$, $y = 0$ при $y \geq 0$.
24. $3z^2 = x^2 + y^2$, $z^2 = 3(x^2 + y^2)$, $x^2 + y^2 + z^2 = 4z$, $x = 0$ при
 $x \geq 0$, $z \geq 0$.
25. $z = 2 - \sqrt{4 - x^2 - y^2}$, $z = 4 - \sqrt{x^2 + y^2}$, $y = 0$ при $y \leq 0$.
26. $z = -\frac{5}{16}(x^2 + y^2)$, $z = -\sqrt{x^2 + y^2 + 9}$, $y = 0$ при $y \leq 0$.
27. $x^2 + y^2 + z^2 = 9$, $x^2 + y^2 + z^2 - 2\sqrt{3}z = 0$ при $x^2 + y^2 + z^2 \geq 9$.
28. $x^2 + y^2 + z^2 = -2z$, $x^2 + y^2 + z^2 = 1$ при $x^2 + y^2 + z^2 \geq 1$.
29. $z = \frac{x^2 + y^2}{4} + 1$, $z = \frac{3(x^2 + y^2)}{4} - 1$, $x = 0$, $y = 0$ при
 $x \geq 0$, $y \geq 0$.
30. $z = 3 - \frac{3(x^2 + y^2)}{4}$, $z = 1 - \frac{x^2 + y^2}{4}$, $x = 0$, $y = 0$ при
 $x \leq 0$, $y \leq 0$.