

Задания (ф.н.п)

1. Найти уравнения касательной плоскости и нормали к заданной поверхности S в точке $M_0(x_0, y_0, z_0)$.

1.1. $S: x^2 + y^2 + z^2 + 6z - 4x + 8 = 0, M_0(2, 1, -1)$.

1.2. $S: x^2 + z^2 - 4y^2 = -2xy, M_0(-2, 1, 2)$.

1.3. $S: x^2 + y^2 + z^2 - xy + 3z = 7, M_0(1, 2, 1)$.

1.4. $S: x^2 + y^2 + z^2 + 6y + 4x = 8, M_0(-1, 1, 2)$.

1.5. $S: 2x^2 - y^2 + z^2 - 4z + y = 13, M_0(2, 1, -1)$.

1.6. $S: x^2 + y^2 + z^2 - 6y + 4z + 4 = 0, M_0(2, 1, -1)$.

1.7. $S: x^2 + z^2 - 5yz + 3y = 46; M_0(1, 2, -3)$.

1.8. $S: x^2 + y^2 - xz - yz = 0, M_0(0, 2, 2)$.

1.9. $S: x^2 + y^2 + 2yz - z^2 + y - 2z = 2, M_0(1, 1, 1)$.

1.10. $S: y^2 - z^2 + x^2 - 2xz + 2x = z, M_0(1, 1, 1)$.

1.11. $S: z = x^2 + y^2 - 2xy + 2x - y, M_0(-1, -1, -1)$.

1.12. $S: z = y^2 - x^2 + 2xy - 3y, M_0(1, -1, 1)$.

1.13. $S: z = x^2 - y^2 - 2xy - x - 2y, M_0(-1, 1, 1)$.

1.14. $S: x^2 - 2y^2 + z^2 + xz - 4y = 13, M_0(3, 1, 2)$.

1.15. $S: 4y^2 - z^2 + 4xy - xz + 3z = 9, M_0(1, -2, 1)$.

1.16. $S: z = x^2 + y^2 - 3xy - x + y + 2, M_0(2, 1, 0)$.

1.17. $S: 2x^2 - y^2 + 2z^2 + xy + xz = 3, M_0(1, 2, 1)$.

1.18. $S: x^2 - y^2 + z^2 - 4x + 2y = 14, M_0(3, 1, 4)$.

1.19. $S: x^2 + y^2 - z^2 + xz + 4y = 4, M_0(1, 1, 2)$.

1.20. $S: x^2 - y^2 - z^2 + xz + 4x = -5, M_0(-2, 1, 0)$.

1.21. $S: x^2 + y^2 - xz + yz - 3x = 11, M_0(1, 4, -1)$.

1.22. $S: x^2 + 2y^2 + z^2 - 4xz = 8, M_0(0, 2, 0)$.

1.23. $S: x^2 - y^2 - 2z^2 - 2y = 0, M_0(-1, -1, 1)$.

1.24. $S: x^2 + y^2 - 3z^2 + xy = -2z, M_0(1, 0, 1)$.

1.25. $S: 2x^2 - y^2 + z^2 - 6x + 2y + 6 = 0, M_0(1, -1, 1)$.

1.26. $S: x^2 + y^2 - z^2 + 6xy - z = 8, M_0(1, 1, 0)$.

1.27. $S: z = 2x^2 - 3y^2 + 4x - 2y + 10, M_0(-1, 1, 3)$.

1.28. $S: z = x^2 + y^2 - 4xy + 3x - 15, M_0(-1, 3, 4)$.

1.29. $S: z = x^2 + 2y^2 + 4xy - 5y - 10, M_0(-7, 1, 8)$.

1.30. $S: z = 2x^2 - 3y^2 + xy + 3x + 1, M_0(1, -1, 2)$.

2. Найти вторые частные производные указанных функций. Убедиться в том, что $z''_{xy} = z''_{yx}$.

2.1. $z = e^{x^2 - y^2}$.

2.2. $z = \text{ctg}(x + y)$.

2.3. $z = \text{tg}(x/y)$.

2.4. $z = \cos(xy^2)$.

2.5. $z = \sin(x^2 - y)$.

2.6. $z = \text{arctg}(x + y)$.

2.7. $z = \arcsin(x - y)$.

2.8. $z = \arccos(2x + y)$.

2.9. $z = \text{arcctg}(x - 3y)$.

2.10. $z = \ln(3x^2 - 2y^2)$.

2.11. $z = e^{2x^2 + y^2}$.

2.12. $z = \text{ctg}(y/x)$.

2.13. $z = \text{tg}\sqrt{xy}$.

2.14. $z = \cos(x^2y^2 - 5)$.

2.15. $z = \sin\sqrt{x^3y}$.

2.16. $z = \arcsin(x - 2y)$.

2.17. $z = \arccos(4x - y)$.

2.18. $z = \text{arctg}(5x + 2y)$.

2.19. $z = \text{arctg}(2x - y)$.

2.20. $z = \ln(4x^2 - 5y^3)$.

2.21. $z = e^{\sqrt{x+y}}$.

2.22. $z = \arcsin(4x + y)$.

2.23. $z = \arccos(x - 5y)$.

2.24. $z = \sin\sqrt{xy}$.

2.25. $z = \cos(3x^2 - y^3)$.

2.26. $z = \text{arctg}(3x + 2y)$.

2.27. $z = \ln(5x^2 - 3y^4)$.

2.28. $z = \text{arcctg}(x - 4y)$.

2.29. $z = \ln(3xy - 4)$.

2.30. $z = \text{tg}(xy^2)$.

3. Проверить, удовлетворяет ли указанному уравнению данная функция u .

$$3.1. \quad x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0, \quad u = \frac{y}{x}.$$

$$3.2. \quad x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3(x^3 - y^3), \quad u = \ln \frac{x}{y} + x^3 - y^3.$$

$$3.3. \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad u = \ln(x^2 + (y+1)^2).$$

$$3.4. \quad y \frac{\partial^2 u}{\partial x \partial y} = (1 + y \ln x) \frac{\partial u}{\partial x}, \quad u = x^y.$$

$$3.5. \quad x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u, \quad u = \frac{xy}{x+y}.$$

$$3.6. \quad x^2 \frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} = 0, \quad u = e^{xy}.$$

$$3.7. \quad a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}, \quad u = \sin^2(x - ay).$$

$$3.8. \quad x^2 \frac{\partial^2 u}{\partial x^2} - y^2 \frac{\partial^2 u}{\partial y^2} = 0, \quad u = y \sqrt{\frac{y}{x}}.$$

$$3.9. \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0, \quad u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

$$3.10. \quad a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}, \quad u = e^{-\cos(x+ay)}.$$

$$3.11. \quad \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0, \quad u = (x-y)(y-z)(z-x).$$

$$3.12. \quad x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = u, \quad u = x \ln \frac{y}{x}.$$

$$3.13. \quad y \frac{\partial u}{\partial x} - x \frac{\partial u}{\partial y} = 0, \quad u = \ln(x^2 + y^2).$$

$$3.14. \quad x^2 \frac{\partial u}{\partial x} - xy \frac{\partial u}{\partial y} + y^2 = 0, \quad u = \frac{y^2}{3x} + \arcsin(xy).$$

$$3.15. \quad x^2 \frac{\partial^2 u}{\partial x^2} - 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + 2xy, \quad u = 0, \quad u = e^{xy}$$

$$3.16. \quad \frac{\partial^2 u}{\partial x \partial y} = 0, \quad u = \operatorname{arctg} \frac{x+y}{1-xy}.$$

$$3.17. \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad u = \ln(x^2 + y^2 + 2x + 1).$$

$$3.18. \quad x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + u = 0, \quad u = \frac{2x+3y}{x^2+y^2}.$$

4. Исследовать на экстремум следующие функции.

- 4.1. $z = y\sqrt{x} - 2y^2 - x + 14y$. (Ответ: $z_{\max}(4, 4) = 28$.)
4.2. $z = x^3 + 8y^3 - 6xy + 5$. (Ответ: $z_{\min}(1; 0,5) = 4$.)
4.3. $z = 1 + 15x - 2x^2 - xy - 2y^2$. (Ответ: $z_{\max}(-4, -1) = -97$.)
4.4. $z = 1 + 6x - x^2 - xy - y^2$. (Ответ: $z_{\max}(4, -2) = 13$.)
4.5. $z = x^3 + y^2 - 6xy - 39x + 18y + 20$. (Ответ: $z_{\min}(5, 6) = -86$.)
4.6. $z = 2x^3 + 2y^3 - 6xy + 5$. (Ответ: $z_{\min}(1, 1) = 3$.)
4.7. $z = 3x^3 + 3y^3 - 9xy + 10$. (Ответ: $z_{\min}(1, 1) = 7$.)
4.8. $z = x^2 + xy + y^2 + x - y + 1$. (Ответ: $z_{\min}(-1, 1) = 0$.)
4.9. $z = 4(x - y) - x^2 - y^2$. (Ответ: $z_{\max}(2, -2) = 8$.)
4.10. $z = 6(x - y) - 3x^2 - 3y^2$. (Ответ: $z_{\max}(1, -1) = 6$.)
4.11. $z = x^2 + xy + y^2 - 6x - 9y$. (Ответ: $z_{\min}(1, 4) = -21$.)
4.12. $z = (x - 2)^2 + 2y^2 - 10$. (Ответ: $z_{\min}(2, 0) = -10$.)
4.13. $z = (x - 5)^2 + y^2 + 1$. (Ответ: $z_{\min}(5, 0) = 1$.)
4.14. $z = x^3 + y^3 - 3xy$. (Ответ: $z_{\min}(1, 1) = -1$.)
4.15. $z = 2xy - 2x^2 - 4y^2$. (Ответ: $z_{\max}(0, 0) = 0$.)
4.16. $z = x\sqrt{y} - x^2 - y + 6x + 3$. (Ответ: $z_{\max}(4, 4) = 15$.)
4.17. $z = 2xy - 5x^2 - 3y^2 + 2$. (Ответ: $z_{\max}(0, 0) = 2$.)
4.18. $z = xy(12 - x - y)$. (Ответ: $z_{\max}(4, 4) = 64$.)
4.19. $z = xy - x^2 - y^2 + 9$. (Ответ: $z_{\max}(0, 0) = 9$.)
4.20. $z = 2xy - 3x^2 - 2y^2 + 10$. (Ответ: $z_{\max}(0, 0) = 10$.)
4.21. $z = x^3 + 8y^3 - 6xy + 1$. (Ответ: $z_{\min}(1; 0,5) = 0$.)
4.22. $z = y\sqrt{x} - y^2 - x + 6y$. (Ответ: $z_{\max}(4, 4) = 12$.)
4.23. $z = x^2 - xy + y^2 + 9x - 6y + 20$. (Ответ: $z_{\min}(-4, 1) = -1$.)
4.24. $z = xy(6 - x - y)$. (Ответ: $z_{\max}(2, 2) = 8$.)
4.25. $z = x^2 + y^2 - xy + x + y$. (Ответ: $z_{\min}(-1, -1) = -1$.)
4.26. $z = x^2 + xy + y^2 - 2x - y$. (Ответ: $z_{\min}(1, 0) = -1$.)
4.27. $z = (x - 1)^2 + 2y^2$. (Ответ: $z_{\min}(1, 0) = 0$.)
4.28. $z = xy - 3x^2 - 2y^2$. (Ответ: $z_{\max}(0, 0) = 0$.)
4.29. $z = x^2 + 3(y + 2)^2$. (Ответ: $z_{\min}(0, -2) = 0$.)
4.30. $z = 2(x + y) - x^2 - y^2$. (Ответ: $z_{\max}(1, 1) = 2$.)

5. Найти наибольшее и наименьшее значения функции $z = z(x, y)$ в области \bar{D} , ограниченной заданными линиями.

5.1. $z = 3x + y - xy$, \bar{D} : $y = x$, $y = 4$, $x = 0$. (Ответ: $z_{\text{наиб}}(2, 2) = 4$, $z_{\text{наим}}(0, 0) = z(4, 4) = 0$.)

5.2. $z = xy - x - 2y$, \bar{D} : $x = 3$, $y = x$, $y = 0$. (Ответ: $z_{\text{наиб}}(0, 0) = z(3, 3) = 0$, $z_{\text{наим}}(3, 0) = -3$.)

5.3. $z = x^2 + 2xy - 4x + 8y$, \bar{D} : $x = 0$, $x = 1$, $y = 0$, $y = 2$. (Ответ: $z_{\text{наиб}}(1, 2) = 17$, $z_{\text{наим}}(1, 0) = -3$.)

5.4. $z = 5x^2 - 3xy + y^2$, \bar{D} : $x = 0$, $x = 1$, $y = 0$, $y = 1$. (Ответ: $z_{\text{наиб}}(1, 0) = 5$, $z_{\text{наим}}(0, 0) = 0$.)

5.5. $z = x^2 + 2xy - y^2 - 4x$, \bar{D} : $x - y + 1 = 0$, $x = 3$, $y = 0$. (Ответ: $z_{\text{наиб}}(3, 3) = 6$, $z_{\text{наим}}(2, 0) = -4$.)

5.6. $z = x^2 + y^2 - 2x - 2y + 8$, D : $x = 0$, $y = 0$, $x + y - 1 = 0$. (Ответ: $z_{\text{наиб}}(0, 0) = 8$, $z_{\text{наим}}(0,5; 0,5) = 6,5$.)

5.7. $z = 2x^3 - xy^2 + y^2$, \bar{D} : $x = 0$, $x = 1$, $y = 0$, $y = 6$. (Ответ: $z_{\text{наиб}}(0, 6) = 36$, $z_{\text{наим}}(0, 0) = 0$.)

5.8. $z = 3x + 6y - x^2 - xy - y^2$, $\bar{D}: x = 0, x = 1, y = 0, y = 1$. (Ответ: $z_{\text{наиб}}(1, 1) = 6, z_{\text{наим}}(0, 0) = 0$.)

5.9. $z = x^2 - 2y^2 + 4xy - 6x - 1$, $\bar{D}: x = 0, y = 0, x + y - 3 = 0$. (Ответ: $z_{\text{наиб}}(0, 0) = -1, z_{\text{наим}}(0, 3) = -19$.)

5.10. $z = x^2 + 2xy - 10$, $\bar{D}: y = 0, y = x^2 - 4$. (Ответ: $z_{\text{наиб}}\left(-\frac{4}{3}, -\frac{2}{5}\right) = -\frac{62}{27}, z_{\text{наим}}(1, -3) = -15$.)

5.11. $z = xy - 2x - y$, $\bar{D}: x = 0, x = 3, y = 0, y = 4$. (Ответ: $z_{\text{наиб}}(3, 4) = 2, z_{\text{наим}}(3, 0) = -6$.)

5.12. $z = \frac{1}{2}x^2 - xy$, $\bar{D}: y = 8, y = 2x^2$. (Ответ: $z_{\text{наиб}}(-2, 8) = 18, z_{\text{наим}}(2, 8) = -14$.)

5.13. $z = 3x^2 + 3y^2 - 2x - 2y + 2$, $\bar{D}: x = 0, y = 0, x + y - 1 = 0$. (Ответ: $z_{\text{наиб}}(0, 1) = z(1, 0) = 3, z_{\text{наим}}\left(\frac{1}{3}, \frac{1}{3}\right) = \frac{4}{3}$.)

5.14. $z = 2x^2 + 3y^2 + 1$, $\bar{D}: y = \sqrt{9 - \frac{9}{4}x^2}, y = 0$. (Ответ: $z_{\text{наиб}}(0, 3) = 28, z_{\text{наим}}(0, 0) = 1$.)

5.15. $z = x^2 - 2xy - y^2 + 4x + 1$, $\bar{D}: x = -3, y = 0, x + y + 1 = 0$. (Ответ: $z_{\text{наиб}}(-3, 2) = 6, z_{\text{наим}}(-2, 0) = -3$.)

5.16. $z = 3x^2 + 3y^2 - x - y + 1$, $\bar{D}: x = 5, y = 0, x - y - 1 = 0$. (Ответ: $z_{\text{наиб}}(5, 4) = 115, z_{\text{наим}}(1, 0) = 3$.)

5.17. $z = 2x^2 + 2xy - \frac{1}{2}y^2 - 4x$, $\bar{D}: y = 2x, y = 2, x = 0$. (Ответ: $z_{\text{наиб}}(0, 0) = z(1, 2) = 0, z_{\text{наим}}(0, 2) = -2$.)

5.18. $z = x^2 - 2xy + \frac{5}{2}y^2 - 2x$, $\bar{D}: x = 0, x = 2, y = 0, y = 2$. (Ответ: $z_{\text{наиб}}(0, 2) = 10, z_{\text{наим}}\left(\frac{5}{3}, \frac{2}{3}\right) = -1,67$.)

5.19. $z = xy - 3x - 2y$, $\bar{D}: x = 0, x = 4, y = 0, y = 4$. (Ответ: $z_{\text{наиб}}(0, 0) = 0, z_{\text{наим}}(4, 0) = -12$.)

5.20. $z = x^2 + xy - 2$, $\bar{D}: y = 4x^2 - 4, y = 0$. (Ответ: $z_{\text{наиб}}\left(-\frac{2}{3}, -2,22\right) = -0,07, z_{\text{наим}}(0,5, -3) = -3,25$.)

5.21. $z = x^2y(4 - x - y)$, $\bar{D}: x = 0, y = 0, y = 6 - x$. (Ответ: $z_{\text{наиб}}(2, 1) = 4, z_{\text{наим}}(4, 2) = -64$.)

5.22. $z = x^3 + y^3 - 3xy$, $\bar{D}: x = 0, x = 2, y = -1, y = 2$. (Ответ: $z_{\text{наиб}}(2, -1) = 13, z_{\text{наим}}(0, -1) = -1$.)

5.23. $z = 4(x - y) - x^2 - y^2$, $\bar{D}: x + 2y = 4, x - 2y = 4, x = 0$. (Ответ: $z_{\text{наиб}}\left(\frac{8}{5}, \frac{6}{5}\right) = \frac{36}{5}, z_{\text{наим}}(0, 2) = -12$.)

5.24. $z = x^2 + 2xy - y^2 - 4x$, $\bar{D}: x = 3, y = 0, y = x + 1$.
(Ответ: $z_{\text{наиб}}(3, 3) = 6, z_{\text{наим}}(2, 0) = -4$.)

5.25. $z = 6xy - 9x^2 - 9y^2 + 4x + 4y$, $\bar{D}: x = 0, x = 1, y = 0, y = 2$. (Ответ: $z_{\text{наиб}}\left(\frac{1}{3}, \frac{1}{3}\right) = \frac{4}{3}, z_{\text{наим}}(0, 2) = -28$.)

5.26. $z = x^2 + 2xy - y^2 - 2x + 2y$, $\bar{D}: y = x + 2, y = 0, x = 2$. (Ответ: $z_{\text{наиб}}(2, 3) = 9, z_{\text{наим}}(1, 0) = -1$.)

5.27. $z = 4 - 2x^2 - y^2$, $\bar{D}: y = 0, y = \sqrt{1 - x^2}$. (Ответ: $z_{\text{наиб}}(0, 0) = 4, z_{\text{наим}}(-1, 0) = z(1, 0) = 2$.)

5.28. $z = 5x^2 - 3xy + y^2 + 4$, $\bar{D}: x = -1, x = 1, y = -1, y = 1$. (Ответ: $z_{\text{наиб}}(-1, 1) = z(1, -1) = 13, z_{\text{наим}}(0, 0) = 4$.)

5.29. $z = x^2 + 2xy + 4x - y^2$, $\bar{D}: x + y + 2 = 0, x = 0, y = 0$. (Ответ: $z_{\text{наиб}}(0, 0) = 0, z_{\text{наим}}(-2, 0) = z(0, -4) = -4$.)

5.30. $z = 2x^2y - x^3y - x^2y^2$, $\bar{D}: x = 0, y = 0, x + y = 6$.
(Ответ: $z_{\text{наиб}}(1, 0, 5) = 0, 25, z_{\text{наим}}(4, 2) = -128$.)

6. Сделать схематический рисунок тела T , ограниченного поверхностями:

1. $z = 2 + \sqrt{4 - x^2 - y^2}$, $z = \sqrt{x^2 + y^2}$, $x = 0$ при $x \leq 0$.

2. $z + 4 = x^2 + y^2$, $9z = 5(x^2 + y^2)$, $y = 0$ при $y \geq 0$.

3. $3z^2 = x^2 + y^2$, $z^2 = 3(x^2 + y^2)$, $x^2 + y^2 + z^2 = 9$ при $z \geq 0$.

4. $x^2 + y^2 = z^2 - 5$, $\frac{z^2}{9} = \frac{x^2 + y^2}{4}$, $x = 0$ при $z \geq 0$, $x \geq 0$.

5. $\frac{z^2}{9} = 1 - \frac{x^2 + y^2}{25}$, $x^2 + y^2 = 9$, $z = 0$, $x = 0$ при $z \geq 0$, $x \leq 0$.

6. $z = 4 - 2\sqrt{x^2 + y^2}$, $z = \frac{x^2 + y^2}{2} - 2$, $y = 0$ при $y \geq 0$.

7. $z = \frac{5}{3}\sqrt{9 - x^2 - y^2}$, $z = \frac{5}{4}\sqrt{x^2 + y^2}$, $x = 0$ при $x \leq 0$.

8. $x^2 + y^2 + z^2 = 4$, $x^2 + y^2 + (z - 2)^2 = 4$ при $0 \leq z \leq 2$.

9. $z = \sqrt{4 - x^2 - y^2}$, $z = \sqrt{x^2 + y^2} - 2$, $x = 0$ при $x \geq 0$.

10. $z = 4 - x^2 - y^2$, $9z = -5(x^2 + y^2)$, $y = 0$ при $y \leq 0$.

11. $z = 2 - \sqrt{x^2 + y^2}$, $z = 4 - x^2 - y^2$, $x = 0$ при $x \leq 0$.

12. $z^2 = 3(x^2 + y^2)$, $x^2 + y^2 + z^2 = 2z$, $x = 0$ при $x \geq 0$,
 $0 \leq z \leq \sqrt{3(x^2 + y^2)}$.
13. $x^2 + y^2 + 4z^2 = 4$, $x^2 + y^2 = 1$, $x = 0$, $z = 0$ при $x^2 + y^2 \geq 1$,
 $x \leq 0$.
14. $2z = 4 - x^2 - y^2$, $z = 2\sqrt{x^2 + y^2} - 4$, $y = 0$ при $y \geq 0$, $z \geq -4$.
15. $z = \sqrt{x^2 + y^2}$, $z + 2 = x^2 + y^2$, $x = 0$ при $x \leq 0$.
16. $x^2 + y^2 + z^2 = 9$, $x^2 + y^2 + z^2 = 2\sqrt{3}z$, $x = 0$, $z = 0$ при $x \geq 0$,
 $0 \leq 2\sqrt{3}z \leq x^2 + y^2 + z^2$.
17. $z = -\sqrt{4 - x^2 - y^2}$, $z = 2 - \sqrt{x^2 + y^2}$, $y = 0$ при $y \geq 0$, $z \leq 2$.
18. $z = x^2 + y^2$, $z = \frac{5}{9}(x^2 + y^2) + 4$, $y = 0$ при $y \leq 0$.
19. $z = 2 - x^2 - y^2$, $z = -\sqrt{x^2 + y^2}$, $x = 0$ при $x \geq 0$.
20. $z^2 = x^2 + y^2$, $x^2 + y^2 + z^2 = 4z$, $x = 0$ при $x \leq 0$,
 $z \geq \sqrt{x^2 + y^2}$.
21. $4x^2 + 4y^2 + z^2 = 36$, $x^2 + y^2 = 4$, $y = 0$, $z = 0$ при
 $y \geq 0$, $z \leq 0$, $x^2 + y^2 \geq 4$.
22. $z = -2\sqrt{x^2 + y^2}$, $2(z + 6) = x^2 + y^2$, $y = 0$ при $y \leq 0$.
23. $z = \sqrt{x^2 + y^2 + 9}$, $4z = 5\sqrt{x^2 + y^2}$, $y = 0$ при $y \geq 0$.
24. $3z^2 = x^2 + y^2$, $z^2 = 3(x^2 + y^2)$, $x^2 + y^2 + z^2 = 4z$, $x = 0$ при
 $x \geq 0$, $z \geq 0$.
25. $z = 2 - \sqrt{4 - x^2 - y^2}$, $z = 4 - \sqrt{x^2 + y^2}$, $y = 0$ при $y \leq 0$.
26. $z = -\frac{5}{16}(x^2 + y^2)$, $z = -\sqrt{x^2 + y^2 + 9}$, $y = 0$ при $y \leq 0$.
27. $x^2 + y^2 + z^2 = 9$, $x^2 + y^2 + z^2 - 2\sqrt{3}z = 0$ при $x^2 + y^2 + z^2 \geq 9$.
28. $x^2 + y^2 + z^2 = -2z$, $x^2 + y^2 + z^2 = 1$ при $x^2 + y^2 + z^2 \geq 1$.
29. $z = \frac{x^2 + y^2}{4} + 1$, $z = \frac{3(x^2 + y^2)}{4} - 1$, $x = 0$, $y = 0$ при
 $x \geq 0$, $y \geq 0$.
30. $z = 3 - \frac{3(x^2 + y^2)}{4}$, $z = 1 - \frac{x^2 + y^2}{4}$, $x = 0$, $y = 0$ при
 $x \leq 0$, $y \leq 0$.