## Задания

*1.* Найти уравнения касательной плоскости и нормали к заданной поверхности $S$ в точке $M_0(x_0,\ y_0,\ z_0)$.

**1.1.** $S$: $x^2+y^2+z^2+6z-4x+8=0$, $M_0(2,\ 1,\ -1)$.

**1.2.** $S$: $x^2+z^2-4y^2=-2xy$, $M_0(-2,\ 1,\ 2)$.

**1.3.** $S$: $x^2+y^2+z^2-xy+3z=7$, $M_0(1,\ 2,\ 1)$.

**1.4.** $S$: $x^2+y^2+z^2+6y+4x=8$, $M_0(-1,\ 1,\ 2)$.

**1.5.** $S$: $2x^2-y^2+z^2-4z+y=13$, $M_0(2,\ 1,\ -1)$.

**1.6.** $S$: $x^2+y^2+z^2-6y+4z+4=0$, $M_0(2,\ 1,\ -1)$.

**1.7.** $S$: $x^2+z^2-5yz+3y=46$, $M_0(1,\ 2,\ -3)$.

**1.8.** $S$: $x^2+y^2-xz-yz=0$, $M_0(0,\ 2,\ 2)$.

**1.9.** $S$: $x^2+y^2+2yz-z^2+y-2z=2$, $M_0(1,\ 1,\ 1)$.

**1.10.** $S$: $y^2-z^2+x^2-2xz+2x=z$, $M_0(1,\ 1,\ 1)$.

**1.11.** $S$: $z=x^2+y^2-2xy+2x-y$, $M_0(-$ [illegible] $-1$ [illegible]

**1.12.** $S$: $z=y^2-x^2+2xy-3y$, $M_0(1,\ -1$ [illegible]

**1.13.** $S$: $z=x^2-y^2-2xy-x-2y$, $M_0(-1,$ [illegible]$)$.

**1.14.** $S$: $x^2-2y^2+z^2+xz-4y=13$, $M_0(3,\ 1,\ 2)$.

**1.15.** $S$: $4y^2-z^2+4xy-xz+3z=9$, $M_0(1,\ -2,\ 1)$.

**1.16.** $S$: $z=x^2+y^2-3xy-x+y+2$, $M_0(2,\ 1,\ 0)$.

**1.17.** $S$: $2x^2-y^2+2z^2+xy+xz=3$, $M_0(1,\ 2,\ 1)$.

**1.18.** $S$: $x^2-y^2+z^2-4x+2y=14$, $M_0(3,\ 1,\ 4)$.

**1.19.** $S$: $x^2+y^2-z^2+xz+4y=4$, $M_0(1,\ 1,\ 2)$.

**1.20.** $S$: $x^2-y^2-z^2+xz+4x=-5$, $M_0(-2,\ 1,\ 0)$.

**1.21.** $S$: $x^2+y^2-xz+yz-3x=11$, $M_0(1,\ 4,\ -1)$.

**1.22.** $S$: $x^2+2y^2+z^2-4xz=8$, $M_0(0,\ 2,\ 0)$.

**1.23.** $S$: $x^2-y^2-2z^2-2y=0$, $M_0(-1,\ -1,\ 1)$.

**1.24.** $S$: $x^2+y^2-3z^2+xy=-2z$, $M_0(1,\ 0,\ 1)$.

**1.25.** $S$: $2x^2-y^2+z^2-6x+2y+6=0$, $M_0(1,\ -1,\ 1)$.

**1.26.** $S$: $x^2+y^2-z^2+6xy-z=8$, $M_0(1,\ 1,\ 0)$.

**1.27.** $S$: $z=2x^2-3y^2+4x-2y+10$, $M_0(-1,\ 1,\ 3)$.

**1.28.** $S$: $z=x^2+y^2-4xy+3x-15$, $M_0(-1,\ 3,\ 4)$.

**1.29.** $S$: $z=x^2+2y^2+4xy-5y-10$, $M_0(-7,\ 1,\ 8)$.

**1.30.** $S$: $z=2x^2-3y^2+xy+3x+1$, $M_0(1,\ -1,\ 2)$.

**2.** Найти вторые частные производные указанных функций. Убедиться в том, что $z''_{xy} = z''_{yx}$.

**2.1.** $z = e^{x^2 - y^2}$. **2.2.** $z = \operatorname{ctg}(x + y)$.

**2.3.** $z = \operatorname{tg}(x/y)$. **2.4.** $z = \cos(xy^2)$.

**2.5.** $z = \sin(x^2 - y)$. **2.6.** $z = \operatorname{arctg}(x + y)$.

**2.7.** $z = \arcsin(x - y)$. **2.8.** $z = \arccos(2x + y)$.

**2.9.** $z = \operatorname{arcctg}(x - 3y)$. **2.10.** $z = \ln(3x^2 - 2y^2)$.

**2.11.** $z = e^{2x^2 + y^2}$. **2.12.** $z = \operatorname{ctg}(y/x)$.

**2.13.** $z = \operatorname{tg}\sqrt{xy}$. **2.14.** $z = \cos(x^2y^2 - 5)$.

**2.15.** $z = \sin\sqrt{x^3y}$. **2.16.** $z = \arcsin(x - 2y)$.

**2.17.** $z = \arccos(4x - y)$. **2.18.** $z = \operatorname{arctg}(5x + 2y)$.

**2.19.** $z = \operatorname{arctg}(2x - y)$. **2.20.** $z = \ln(4x^2 - 5y^3)$.

**2.21.** $z = e^{\sqrt{x + y}}$. **2.22.** $z = \arcsin($



**2.23.** $z = \arccos(x - 5y)$. **2.24.** $z = \sin\sqrt{xy}$.

**2.25.** $z = \cos(3x^2 - y^3)$. **2.26.** $z = \operatorname{arctg}(3x + 2y)$.

**2.27.** $z = \ln(5x^2 - 3y^4)$. **2.28.** $z = \operatorname{arcctg}(x - 4y)$.

**2.29.** $z = \ln(3xy - 4)$. **2.30.** $z = \operatorname{tg}(xy^2)$.

*3.* Проверить, удовлетворяет ли указанному уравнению данная функция $u$.

**3.1.** $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=0,\ u=\frac{y}{x}.$

**3.2.** $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3(x^3-y^3),\ u=\ln\frac{x}{y}+x^3-y^3.$

**3.3.** $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,\ u=\ln(x^2+(y+1)^2).$

**3.4.** $y\frac{\partial^2 u}{\partial x\partial y}=(1+y\ln x)\frac{\partial u}{\partial x},\ u=x^y.$

**3.5.** $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2u,\ u=\frac{xy}{x+y}.$

**3.6.** $x^2\frac{\partial^2 u}{\partial x^2}+y^2\frac{\partial^2 u}{\partial y^2}=0,\ u=e^{xy}.$

**3.7.** $a^2\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial y^2},\ u=\sin^2(x-ay).$

**3.8.** $x^2\frac{\partial^2 u}{\partial x^2}-y^2\frac{\partial^2 u}{\partial y^2}=0,\ u=y\sqrt{\frac{y}{x}}.$

**3.9.** $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0,\ u=\frac{1}{\sqrt{x^2+y^2+z^2}}$

**3.10.** $a^2\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial y^2},\ u=e^{-\cos(x+ay)}.$

**3.11.** $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0,\ u=(x-y)(y-z)(z-x).$

**3.12.** $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=u,\ u=x\ln\frac{y}{x}.$

**3.13.** $y\frac{\partial u}{\partial x}-x\frac{\partial u}{\partial y}=0,\ u=\ln(x^2+y^2).$

**3.14.** $x^2\frac{\partial u}{\partial x}-xy\frac{\partial u}{\partial y}+y^2=0,\ u=\frac{y^2}{3x}+\arcsin(xy).$

**3.15.** $x^2\frac{\partial^2 u}{\partial x^2}-2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}+2xy,\ u=0,\ u=e^{xy}$

**3.16.** $\frac{\partial^2 u}{\partial x\partial y}=0,\ u=\operatorname{arctg}\frac{x+y}{1-xy}.$

**3.17.** $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,\ u=\ln(x^2+y^2+2x+1).$

**3.18.** $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+u=0,\ u=\frac{2x+3y}{x^2+y^2}.$

**4.** Исследовать на экстремум следующие функции.

**4.1.** $z = y\sqrt{x} - 2y^2 - x + 14y$. (*Ответ:* $z_{\max}(4,\ 4) = 28$.)

**4.2.** $z = x^3 + 8y^3 - 6xy + 5$. (*Ответ:* $z_{\min}(1;\ 0{,}5) = 4$.)

**4.3.** $z = 1 + 15x - 2x^2 - xy - 2y^2$. (*Ответ:* $z_{\max}(-4,\ -1) = -97$.)

**4.4.** $z = 1 + 6x - x^2 - xy - y^2$. (*Ответ:* $z_{\max}(4,\ -2) = 13$.)

**4.5.** $z = x^3 + y^2 - 6xy - 39x + 18y + 20$. (*Ответ:* $z_{\min}(5,\ 6) = -86$.)

**4.6.** $z = 2x^3 + 2y^3 - 6xy + 5$. (*Ответ:* $z_{\min}(1,\ 1) = 3$.)

**4.7.** $z = 3x^3 + 3y^3 - 9xy + 10$. (*Ответ:* $z_{\min}(1,\ 1) = 7$.)

**4.8.** $z = x^2 + xy + y^2 + x - y + 1$. (*Ответ:* $z_{\min}(-1,\ 1) = 0$.)

**4.9.** $z = 4(x - y) - x^2 - y^2$. (*Ответ:* $z_{\max}(2,\ -2) = 8$.)

**4.10.** $z = 6(x - y) - 3x^2 - 3y^2$. (*Ответ:* $z_{\max}(1,\ -1) = 6$.)

**4.11.** $z = x^2 + xy + y^2 - 6x - 9y$. (*Ответ:* $z_{\min}(1,\ 4) = -21$.)

**4.12.** $z = (x - 2)^2 + 2y^2 - 10$. (*Ответ:* $z_{\min}(2,\ 0) = -10$.)

**4.13.** $z = (x - 5)^2 + y^2 + 1$. (*Ответ:* $z_{\min}(5,\ 0) = 1$.)

**4.14.** $z = x^3 + y^3 - 3xy$. (*Ответ:* $z_{\min}(1,\ 1) = -1$.)

**4.15.** $z = 2xy - 2x^2 - 4y^2$. (*Ответ:* $z_{\max}(0,\ 0) = 0$.)

**4.16.** $z = x\sqrt{y} - x^2 - y + 6x + 3$. (*Ответ:* $z_{\max}(4,\ 4) = 15$.)

**4.17.** $z = 2xy - 5x^2 - 3y^2 + 2$. (*Ответ:* $z_{\max}(0,\ 0) = 2$.)

**4.18.** $z = xy(12 - x - y)$. (*Ответ:* $z_{\max}(4,\ 4) = 64$.)

**4.19.** $z = xy - x^2 - y^2 + 9$. (*Ответ:* $z_{\max}(0,\ 0) = 9$.)

**4.20.** $z = 2xy - 3x^2 - 2y^2 + 10$. (*Ответ:* $z_{\max}(0,\ 0) = 10$.)

**4.21.** $z = x^3 + 8y^3 - 6xy + 1$. (*Ответ:* $z_{\min}(1;\ 0{,}5) = 0$.)

**4.22.** $z = y\sqrt{x} - y^2 - x + 6y$. (*Ответ:* $z_{\max}(4,\ 4) = 12$.)

**4.23.** $z = x^2 - xy + y^2 + 9x - 6y + 20$. (*Ответ:* $z_{\min}(-4,\ 1) = -1$.)

**4.24.** $z = xy(6 - x - y)$. (*Ответ:* $z_{\max}(2,\ 2) = 8$.)

**4.25.** $z = x^2 + y^2 - xy + x + y$. (*Ответ:* $z_{\min}(-1,\ -1) = -1$.)

**4.26.** $z = x^2 + xy + y^2 - 2x - y$. (*Ответ:* $z_{\min}(1,\ 0) = -1$.)

**4.27.** $z = (x - 1)^2 + 2y^2$. (*Ответ:* $z_{\min}(1,\ 0) = 0$.)

**4.28.** $z = xy - 3x^2 - 2y^2$. (*Ответ:* $z_{\max}(0,\ 0) = 0$.)

**4.29.** $z = x^2 + 3(y + 2)^2$. (*Ответ:* $z_{\min}(0,\ -2) = 0$.)

**4.30.** $z = 2(x + y) - x^2 - y^2$. (*Ответ:* $z_{\max}(1,\ 1) = 2$.)

**5.** Найти наибольшее и наменьшее значения функции $z = z(x, y)$ в области $\bar{D}$, ограниченной заданными линиями.

**5.1.** $z = 3x + y - xy$, $\bar{D}$: $y = x$, $y = 4$, $x = 0$. (*Ответ*: $z_{\text{наиб}}(2, 2) = 4$, $z_{\text{наим}}(0, 0) = z(4, 4) = 0$.)

**5.2.** $z = xy - x - 2y$, $\bar{D}$: $x = 3$, $y = x$, $y = 0$. (*Ответ*: $z_{\text{наиб}}(0, 0) = z(3, 3) = 0$, $z_{\text{наим}}(3, 0) = -3$.)

**5.3.** $z = x^2 + 2xy - 4x + 8y$, $\bar{D}$: $x = 0$, $x = 1$, $y = 0$, $y = 2$. (*Ответ*: $z_{\text{наиб}}(1, 2) = 17$, $z_{\text{наим}}(1, 0) = -3$.)

**5.4.** $z = 5x^2 - 3xy + y^2$, $\bar{D}$: $x = 0$, $x = 1$, $y = 0$, $y = 1$. (*Ответ*: $z_{\text{наиб}}(1, 0) = 5$, $z_{\text{наим}}(0, 0) = 0$.)

**5.5.** $z = x^2 + 2xy - y^2 - 4x$, $\bar{D}$: $x - y + 1 = 0$, $x = 3$, $y = 0$. (*Ответ*: $z_{\text{наиб}}(3, 3) = 6$, $z_{\text{наим}}(2, 0) = -4$.)

**5.6.** $z = x^2 + y^2 - 2x - 2y + 8$, $\bar{D}$: $x = 0$, $y = 0$, $x + y - 1 = 0$. (*Ответ*: $z_{\text{наиб}}(0, 0) = 8$, $z_{\text{наим}}(0{,}5;\ 0{,}5) = 6{,}5$.)

**5.7.** $z = 2x^3 - xy^2 + y^2$, $\bar{D}$: $x = 0$, $x = 1$, $y = 0$, $y = 6$. (*Ответ*: $z_{\text{наиб}}(0, 6) = 36$, $z_{\text{наим}}(0, 0) = 0$.)